

Homework Assignment 18: Due at the beginning of class 11/7/01

In Questions 1, the function f will always refer to the function defined by the following equation:

$$f(x) = x^3 - 5x + 1.$$

1. Find an equation for the derivative function $f'(x)$. (Note that you do *not* have to use the limit definition of the derivative here.) Use the equation for $f'(x)$ to find the places where f has a local maximum (top of a hill) or local minimum (bottom of a valley). Briefly explain your reasoning process.

You have probably noticed that the squirrels that live in Harvard Yard are much more active now that when you first arrived here in August. The obvious reason is that the squirrels are attempting to store body fat and food for the lean months of Winter that lie ahead. In Questions 2-5 you will use calculus to study the foraging habits of a ground squirrel and learn how to handle maximization/minimization problems that are not as convenient as Question 1.



Figure 1: The golden-mantled ground squirrel (*Spermophilus saturatus*)

The golden-mantled ground squirrel (*Spermophilus saturatus* - see Figure 1¹) is a small mammal that is common in many parts of North America, especially the Pacific north-west. These animals live in underground burrows. Typically, a ground squirrel spends about 16.5 hours underground and about 7.5 hours on the surface. The squirrels only forage for food when they are on the surface.

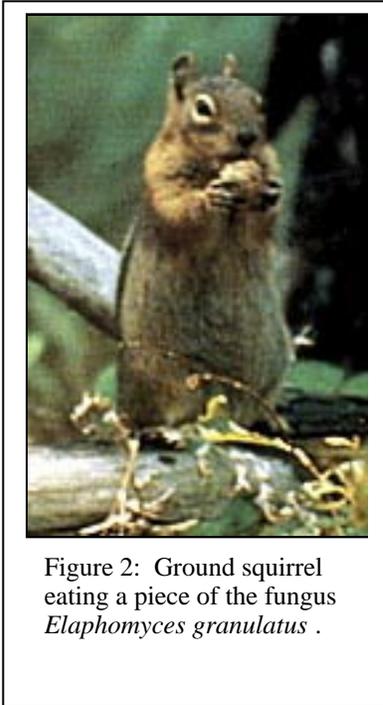
The diet of the golden-mantled ground squirrel includes leaves, bark,

berries and seeds. Two of the most important food items are seeds (normally from pine and fir trees) and fungi (principally *Elaphomyces granulatus* - see Figure 2²).

The foraging habits and energy requirements of this squirrel have been studied to some extent³. The two scientific papers that the rest of this homework assignment are based on are:

¹ Image source: <http://www.washington.edu/burkemuseum/>

² Image source: <http://www.publish.csiro.au/ecos/Ecos91/Ecos91B.htm>



- S.J. Cork and G.J. Kenagy. “Nutritional value of a hypogeous fungus for a forest-dwelling ground squirrel.” *Ecology*, **70**(3): 577-586, 1989.
- G. J. Kenagy and D.F. Hoyt. “Speed and time-energy budget for locomotion in golden-mantled ground squirrels.” *Ecology*, **70**(6): 1834-1839, 1989.

These scientists found that the amount of energy expended by a squirrel depended on whether it was foraging or resting. The average amount of energy that ground squirrels expend⁴ (each hour) while either foraging or resting is shown in Table 1 below.

Activity	Energy expenditure (kJ per hour)
Foraging	13.695
Resting	2.73

Table 1: Average energy expenditures of ground squirrels

Two of the most important food sources for golden-mantled ground squirrels are seeds (from pine or fir) and fungus. The nutritional information, as well as the average amount of time required to find and consume these foods are shown in Table 2 below.

Food	Energy (kJ)	% of energy digested	Search time (minutes)	Consumption time (minutes)
Seeds	26.05	96	18	15
Fungus	17.3	52.2	10	3

Table 2: Nutritional and foraging information for ground squirrel common foods

2. Suppose that T represents the number of hours that a ground squirrel spends foraging each day. Find an equation that gives the energy expended by a squirrel as a function of T . What is the problem domain of your function?
3. As in Question 2, let T represent the number of hours that a ground squirrel spends foraging each day. Find an equation that represents the energy gained by a squirrel

³ The golden-mantled ground squirrel enters a state of near hibernation during the winter, where it dramatically lowers its metabolism and lives on stored body fat. Exactly how the squirrel does this is of interest to researchers who hope to be able to find a way for humans to enter a similar state of “suspended animation.” Two applications of this technology would include “freezing” astronauts for very long-duration space missions (e.g. to Mars) and temporarily suspending the death of critically injured people until they could be moved to suitable medical facilities.

⁴ The numbers given here apply to an animal with a body mass of 150g, which is average for *Spermophilus saturatus*.

that forages in a home range where fungus is the main food, and an equation that represents the energy gained by a squirrel that forages in a home range where seeds⁵ are the main source of food.

4. Depending on the time of the year, the foraging habits of the squirrel can change. During Summer, the squirrels usually only forage for long enough to satisfy their immediate energy needs. However, in the Fall and Spring, the squirrels try to gain as much energy as they possibly can. Calculate the number of hours that a squirrel would be expected to spend foraging in each of the four cases listed below. In each case, show your calculation.

Case 1: It is Summer and the squirrel's home range offers mainly fungus to eat.

Case 2: It is Fall and the squirrel's home range offers mainly fungus to eat. (Remember the problem domain.)

Case 3: It is Summer and the squirrel's home range offers mainly seeds to eat.

Case 4: It is Fall and the squirrel's home range offers mainly seeds to eat. (Remember the problem domain.)

5. Select one type of squirrel diet (fungus OR seeds). Using one set of axes, draw graphs showing:
- the energy expended by the squirrel as a function of the number of hours spent foraging
 - the energy gained by the squirrel as a function of the number of hours spent foraging.

On your graphs, mark the locations of the two points that you calculated in Question 4. Based on what you have seen in Question 1 and Question 5, complete the following phrase:

"If x is restricted to an interval $[a, b]$, then the maximum and minimum values of a function $f(x)$ occur either:

- at a point where $f'(x) = 0$, or,
- at _____ .

⁵ Note the length of time that squirrels must spend consuming seeds. This is because the seeds of pine and fir must first be extracting from cones. This operation takes the squirrels a fair amount of time.