Math X Problem Set #2 Assigned Wednesday, Sept. 27th

First, here are the
Friday Problems: (originally assigned on Monday, Sept. 25)
(Section 1.2 #11 and 29)

#11) (a) $A(4)$ is the area of a 4-sided regular polygon - i.e. a square

quick way to get area:

square can be broken into 4 equal Triangles 

each of area
$\frac{1}{2}$ base $\times$ height $= \frac{1}{2}$

so total area $= 4 \cdot \frac{1}{2} = 2$, so $A(4) = 2$

(b) Yes, $A(n)$ is a well defined function - for each (sensible) input there is a unique output

(c) n needs to be an integer (counting number), bigger than 2 for $A(n)$ to make sense, so the domain can be written as "all integers greater than or equal to 3," or as $\{x \in \mathbb{Z},\ x \geq 3\}$

(d) as n increases you can check that the inscribed polygons use up more of the area of the circle, so yes $A(n)$ is an increasing function
(Note - the question does not ask for a rigorous proof - just your intuition!)

(e) Well, the areas can never get larger than the circle's area, which is just πr^2 with r, the radius, $= 1$, so its area is just π. As the polygons get more sides, they will eventually fill up the circle's area as close to the circle's perimeter (circumference) as you'd like, so the least such upper bound number must just be π

#29) (a) i. $A(4) = 4^2 = 16$ ii. $A(w) = w^2$

iii. $A(\sqrt{2}+3) = (\sqrt{2}+3)^2$ (this is fine, since it doesn't ask you to expand your answer, but it also equals $11 + 6\sqrt{2}$)

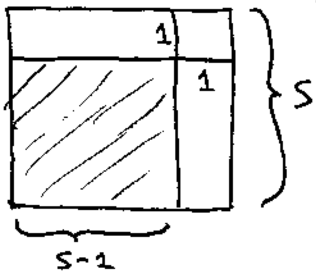
iv. $A(4+h) = (4+h)^2 = 16 + 8h + h^2$

v. $A(x-1) = (x-1)^2 = x^2 - 2x + 1$

(b) $A(s-1) = (s-1)^2 = s^2 - 2s + 1$, whereas $A(s) - 1$ equals $s^2 - 1$. If s is bigger than 1,

#29) continued, then $-2S+1 < -1$ (Why? check by adding $2S$ to both sides: $1 \overset{?}{<} -1+2S$, Then add 1 to both sides: $2 \overset{?}{<} 2S$, and yes this is true if $S>1$)
So $A(s-1) = S^2-2S+1 < A(S)-1 = S^2-1$, as long as $S>1$

(c) In fact the picture gives a nice visual proof of this fact, for if S is bigger than 1, then $S-1$ is bigger than zero, so the shaded part of the square is $A(S-1)$ (the area of a square of side $S-1$). Now note that if you calculate $A(S)-1$, this is equivalent to the area of the big square minus the small square of side 1 in the upper right corner, and this clearly is larger than $A(S-1)$, the shaded area.

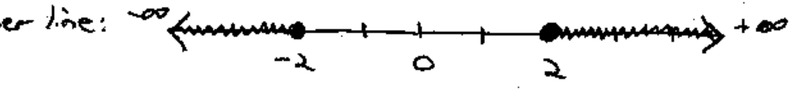


Regular Assignment:
Section 1.3 #3, 5, 7(a), 23, 30 and 41 and Section 2.1 #2, 3, 6(a-d)

§1.3 #3) (a) domain of $f(x) = \sqrt{x}$ is all positive numbers (including zero), since (without considering complex numbers, which are a whole other story!) the square root function is not defined for negative numbers
you could also write domain $= \{x \in \mathbb{R},\ x \geq 0\}$

(b) Now we need $x-3$ to be non-negative, so here the domain is $x \geqslant 3$ or "x is in $[3, +\infty)$"

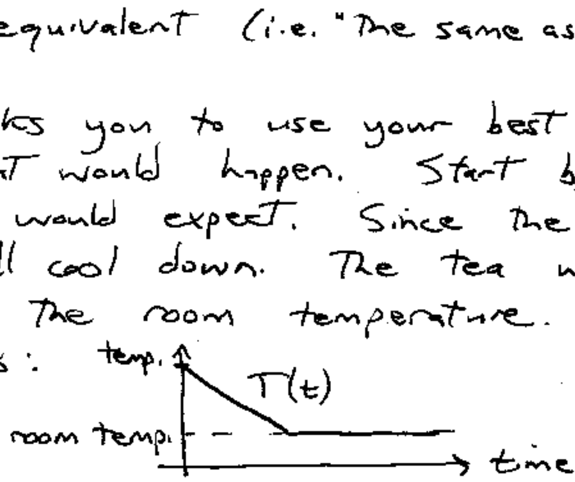
(c) Now we need to determine when $x^2-4 \geqslant 0$, or when is $x^2 \geq 4$. Now x could either be $\geqslant 2$, or $x \leqslant -2$ (either way x^2 will be $\geqslant 4$)
To write this you have, as before, many options!
domain = $x \geqslant 2$ or $x \leqslant -2$
or x is in $(-\infty, -2] \cup [2, +\infty)$
or number line:



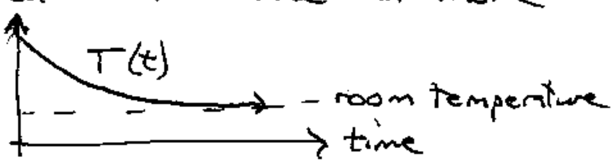
#5) If you think of the range as all possible output values, then all three functions could equal any positive number including zero, so all three have range: $x \geqslant 0$

#7(a) $f(x) = -2x^2$ $\quad g(w) = (-2w)^2 = 4w^2 \neq f(w) = -2w^2$
(I just wrote $f(x)$ out as $f(w)$ to make the comparison easier)
$i(t) = 2(-t^2) = -2t^2$ This <u>is</u> the same as f
$j(x) = -\sqrt{2x^2}\sqrt{x^2}\sqrt{2} = -\sqrt{2}\cdot\sqrt{2}\cdot\sqrt{x^2}\cdot\sqrt{x^2} = -2x^2$
This is also equivalent (i.e. "the same as") function f

#23) This question asks you to use your best judgment, your intuition about what would happen. Start by describing in words what you would expect. Since the tea starts off hot - it will cool down. The tea will not get any cooler than the room temperature. So one possible graph is:

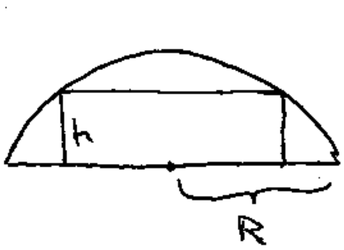


This might not seem right, though, for the tea's temperature to cool down at exactly the same rate over time, then suddenly stop cooling off as it hits room temperature. It seems likely that the tea will settle in more gently to room temperature, more like:

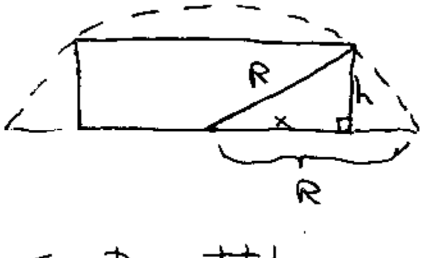


Incidently we'll study more graphs like this next semester when we investigate differential equations!

#30)



(a) to figure out the rectangle's area we need to know its length as well as its height. We can find this out with the same pythagorean trick we've seen in some other examples (look at example 1.10 on page 28)



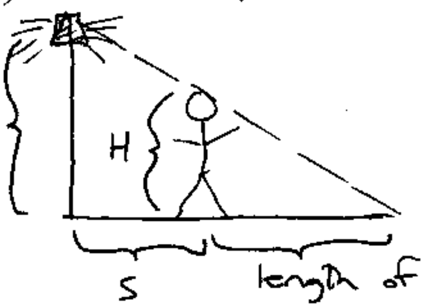
According to the picture, $\frac{1}{2}$ the length of the rectangle, call this x, satisfies $x^2 + h^2 = R^2$
so $x^2 = R^2 - h^2$, or $x = \sqrt{R^2 - h^2}$

so the total area of the rectangle $f(h) = \text{base} \times \text{height} = 2\left(\sqrt{R^2 - h^2}\right)h$

#30) continued (b) and the perimeter is just 2·base + 2·height

or $P = g(h) = 4\cdot\sqrt{R^2-h^2} + 2h$

#41) Draw a picture!



We want to determine the length of his shadow, L, based on the distance, s, he stands away from the lamp post.

First, let's simplify the picture a bit:

Similar triangles yields that $\dfrac{H}{14} = \dfrac{L}{L+S}$ (similar ratios of little to big triangles)

so cross-multiplying: $H(L+S) = 14L$

or $H\cdot L + H\cdot S = 14L$

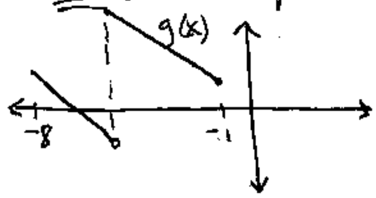
now solve for L: so $H\cdot S = 14\cdot L - H\cdot L = L\cdot(14-H)$

so $L = \dfrac{H\cdot S}{14-H}$, so as a function of s, the distance from lamp post, L, the length of his shadow is given by $L(s) = \dfrac{H\cdot s}{14-H}$

Section 2.1

#2) If f is decreasing the whole time between -8 and -1, then almost by definition the place where it must take on its largest value is at $x=-8$.

[This doesn't depend on continuity, for instance



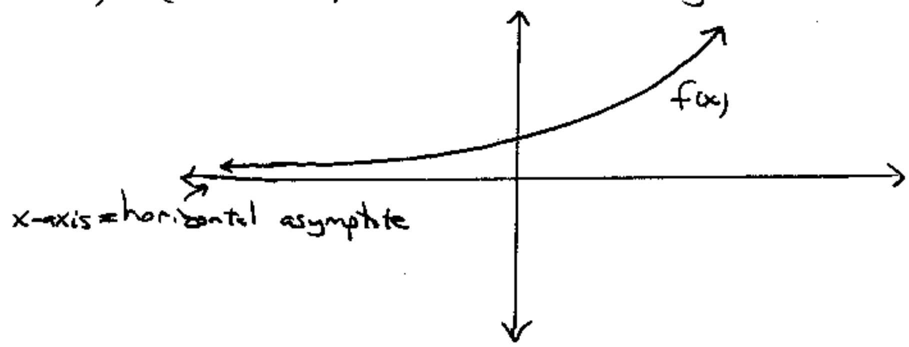
g(x) might look like it fits the description, but $g(x_2)$ is not always $< g(x_1)$ for $x_2 > x_1$ (check definition on page 55)]

Section 2.1 continued

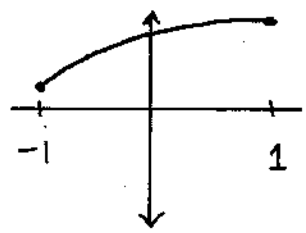
#3) (a) sure, $A(x)$ is a function, it's well defined - you can calculate the area given for any x in $[-2, 8]$

(b) since the shaded area simply gets larger and larger as x goes from -2 to 8, the function $A(x)$ must be an increasing function.

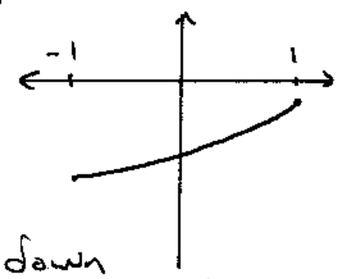
#6) (a) f positive, increasing, concave up:



(b) f positive, increasing, concave down, not possible for all of $\mathbb{R}$, the real numbers, so:



(c) also f negative, increasing, concave up is not possible for all of $\mathbb{R}$, so try



(d) f negative, increasing and concave down is very similar to (a), just flipped over and reversed!