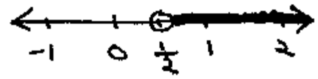
PS #3 Assigned 9/29/00 2.2 #6, 8, 14, 20
2.3 #2, 3, 11, 12

2.2 6 a) $-2x - 7 < -8$

$-2x < -1$

$\boxed{x > \frac{1}{2}}$

or $(\frac{1}{2}, +\infty)$



b) $|-2x - 8| \geq 2$

If $-2x - 8 > 0$

$-2x > 8$

$x < -4$:

$-2x - 8 \geq 2$

$-2x \geq 10$

$x \leq -5$

If $-2x - 8 < 0$

$-2x < 8$

$x > -4$:

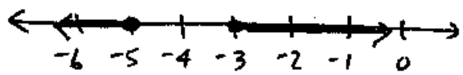
$-(-2x - 8) \geq 2$

$2x + 8 \geq 2$

$2x \geq -6$

$x \geq -3$

So, for $x < -4$, $x \leq -5$ solves the inequality
and for $x > -4$, $x \geq -3$ solves the inequality



8 a) true
b) true
c) not always true
counterexample: $x = 3$, $y = -2$

$|x - y| = |3 - (-2)| = |5| = 5$

$|x| - |y| = |3| - |-2| = 3 - 2 = 1$

$|x - y| \neq |x| - |y|$

d) not always true
counterexample: $x = 3$, $y = -2$

$|x + y| = |3 + (-2)| = |1| = 1$

$|x| + |y| = |3| + |-2| = 3 + 2 = 5$

$|x + y| \neq |x| + |y|$

e) true
f) true

(14) a) $f(x) = 2x^3 + 3x$ is <u>odd</u>.

For odd functions: $f(-x) = -f(x)$

$f(-x) = 2(-x)^3 + 3(-x) = -2x^3 - 3x = -(2x^3 + 3x) = -f(x)$

b) $g(x) = 2x^3 + 3x + 1$ is <u>neither</u>

For odd fxns: $f(-x) = -f(x)$

$f(-x) = 2(-x)^3 + 3(-x) + 1 = -2x^3 - 3x + 1 \neq -f(x)$

For even fxns: $f(-x) = f(x)$

$f(-x) = -2x^3 - 3x + 1 \neq f(x)$

(20) A fxn can be both even and odd.

For a fxn to be both even and odd, it must satisfy the following condition:

$$f(-x) = \underset{\text{odd}}{-f(x)} = \underset{\text{even}}{f(x)}$$

For $f(x)$ to be equal to its negative $(-f(x))$, $f(x)$ must equal 0.

$\boxed{f(x) = 0}$ only fxn that is both even and odd.

$\therefore$ I cannot give 2 examples.

2.3 (2) a) $p(50) - p(20)$

b) $\dfrac{p(50) - p(20)}{p(20)} \times 100$

c) $\dfrac{p(50) - p(20)}{30}$

③ a) $f(c) < f(d) < f(b) < f(a)$

b) $f(b) - f(a) < f(d) - f(c) < f(b) - f(c)$

c) $\dfrac{f(b)-f(a)}{b-a} \Rightarrow$ $f(b) < f(a)$ so $f(b)-f(a) < 0$
$b > a$ so $b-a > 0$
$\therefore$ expression is <u>negative</u>
$\left(\dfrac{\text{negative \#}}{\text{positive \#}} = \text{negative \#} \right)$

$\dfrac{f(a)-f(b)}{a-b} \Rightarrow$ If you multiply both top and bottom by -1, you see that this expression is essentially the same as the first expression and $\therefore$ is negative

$\dfrac{f(c)-f(b)}{c-b} \Rightarrow$ $f(c) < f(b)$ $\therefore$ $f(c)-f(b) < 0$
$c > b$ $\therefore$ $c-b > 0$
this expression is <u>negative</u>.
You can get the relative magnitude of this expression compared to the first 2 by remembering that it represents the average slope between points c and b. From the graph, you can see the average slope between c and b is $<$ the ave. slope between a and b. $\therefore$ the magnitude of this expression is $<$ the 1st 2 expressions.

$\dfrac{f(d)-f(c)}{d-c} \Rightarrow$ positive $\dfrac{f(d)-f(b)}{d-b} \Rightarrow$ negative

$$\boxed{\frac{f(b)-f(a)}{b-a} = \frac{f(a)-f(b)}{a-b} < \frac{f(c)-f(b)}{c-b} < \frac{f(d)-f(b)}{d-b} < \frac{f(d)-f(c)}{d-c}}$$

(11) $$g(t) = \frac{t}{t^2+2} + 3t$$

$$g(-1) = -\frac{1}{3} - 3 = -\frac{10}{3}$$

$$g(1) = \frac{1}{3} + 3 = \frac{10}{3}$$

$$[-1,1] \Rightarrow \frac{\Delta g(t)}{\Delta t} = \frac{\frac{10}{3} - \left(-\frac{10}{3}\right)}{2} = \boxed{\frac{10}{3}}$$

$$g(0) = 0$$

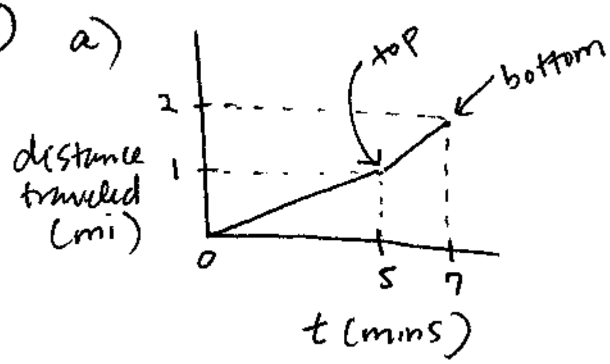
$$g(2) = \frac{2}{6} + 6 = \frac{19}{3}$$

$$[0,2] \Rightarrow \frac{\Delta g(t)}{\Delta t} = \frac{\frac{19}{3} - 0}{2} = \boxed{\frac{19}{6}}$$

$$[1,1+p] \Rightarrow \frac{\Delta g(t)}{\Delta t} = \frac{\frac{1+p}{(1+p)^2+2} + 3(1+p) - \left(\frac{1}{1^2+2} + 3(1)\right)}{\cancel{1}+p-\cancel{1}}$$

$$\boxed{\frac{\Delta g(t)}{\Delta t} = \frac{1}{p}\left[\frac{1+p}{(1+p)^2+2} + 3(1+p) - \frac{10}{3}\right]}$$

(12) a)



Climb:
1mi at 12mph

$x = 1\text{mi} \quad v = 12\text{mph}$

$$v = \frac{x}{t} \Rightarrow t = \frac{x}{v}$$

$$t = \frac{1\text{mi}}{12\text{mi}/\text{h}} = \frac{1}{12}\text{h} = 5\text{mins.}$$

Descent:

$x = 1\text{mi} \quad v = 30\text{mph}$

$$t = \frac{1\text{mi}}{30\text{mi}/\text{h}} = \frac{1}{30}\text{h} = 2\text{mins}$$

b) Ave speed = $\frac{\text{distance}}{\text{time}}$

$$\text{Ave speed} = \frac{2\text{mi}}{7\text{min}}\left(\frac{60\text{min}}{1\text{h}}\right) = \boxed{17.143\text{ mph}} \neq \frac{(12+30)\text{mph}}{2} = 21\text{mph}$$

Not equal to average of 12 and 30 mph, because the cyclist spent a longer time traveling at the slower speed.