Solution Set 13

7.3/ 2. $\lim_{x\to 3} \frac{2x^3-8x^2+5x+3}{x-3}$. Ideally, we'd like for there to be a factor of $x-3$ in the top, so we could cancel off this nasty denominator. So let's see if we can factor it:

$$\lim_{x\to 3}\left[\frac{2x^3-8x^2+5x+3}{x-3}\right] = \lim_{x\to 3}\left[\frac{(x-3)(2x^2-2x-1)}{x-3}\right] = \lim_{x\to 3}\left[2x^2-2x-1\right]$$

$$= 2\cdot 3^2 - 2\cdot 3 - 1 = 18-6-1 = \boxed{11}.$$

#1 a) $h'(t) = \lim_{R\to 0}\left[\frac{h(t+R)-h(t)}{R}\right] = \lim_{R\to 0}\left[\frac{kf(t+R)-kf(t)}{R}\right] =$

$$\lim_{R\to 0}\left[k\left(\frac{f(t+R)-f(t)}{R}\right)\right] = k\lim_{R\to 0}\left[\frac{f(t+R)-f(t)}{R}\right] = \boxed{kf'(t)}$$

b) Since h is just a stretched f, this sort of makes sense:

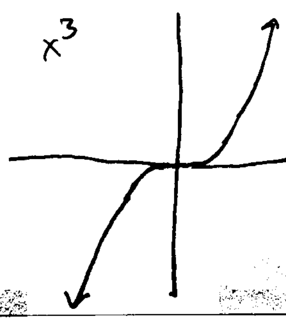
f:

h:

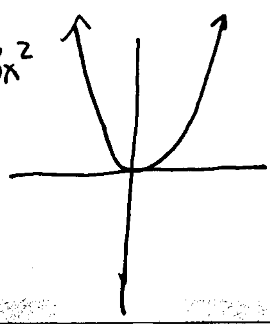
c) To go twice the distance in the same time, ya gotta run twice as fast. Running is the rate of change of position, so the derivative must be twice as big to go twice as far. For k times the distance, one must go k times faster, so $h'(t) = kf'(t)$.

3. a) Since if $h(x) = kg(x)$, then $h'(x) = kg'(x)$, and since $j(x) = h(x)+g(x)$ tells us $j'(x) = h'(x)+g'(x)$, we can compute $\frac{d}{dx}x^3$, $\frac{d}{dx}x^2$, and $\frac{d}{dx}x$ to figure out $f'(x)$. $\frac{d}{dx}x^3 = 3x^2$, $\frac{d}{dx}x^2 = 2x$, $\frac{d}{dx}x = 1$ (this just using the lim definition of the derivative). So $f'(x) = a(3x^2)+b(2x)+c(1)+0 = \boxed{3ax^2+2bx+c}$

b) x^3



$3x^2$



For x^3, the slope is always positive, it goes to 0 at (0,0), and if you draw some tangent lines, the values match up.

7.4/ 4. $|h(x)| \leq 3$ for all x. What is $\lim_{x\to\infty} \frac{h(x)}{x}$.

By the absolute value relation they give us, we know $\frac{-3}{x} \leq \frac{h(x)}{x} \leq \frac{3}{x}$

So $\lim_{x\to\infty} \frac{-3}{x} \leq \lim_{x\to\infty} \frac{h(x)}{x} \leq \lim_{x\to\infty} \frac{3}{x}$. But $\lim_{x\to\infty} \frac{3}{x}$ is 0, as is $\lim_{x\to\infty} \frac{-3}{x}$. So $\lim_{x\to\infty} \frac{h(x)}{x}$, trapped in between with nowhere to go, must also be $\boxed{0}$.

5. $f(x) = \frac{x^2-4}{x+2}$. You'd be tempted to just cancel and say $f(x) = x-2$, but don't forget when $x=-2$, the denominator $= 0 =$ bad. So there's a discontinuity there. Otherwise $f(x)$ looks just like $x-2$ So we can fix him by redefining $f(-2) = -4$.

21.