Solution Set 16

8.1/ 1. $f(x) = x^{1/2}$, so the slope of the tangent line at $x=25$ is $f'(25) = \frac{1}{2}(x^{-1/2})@25 = \frac{1}{2\sqrt{25}} = \frac{1}{10}$.

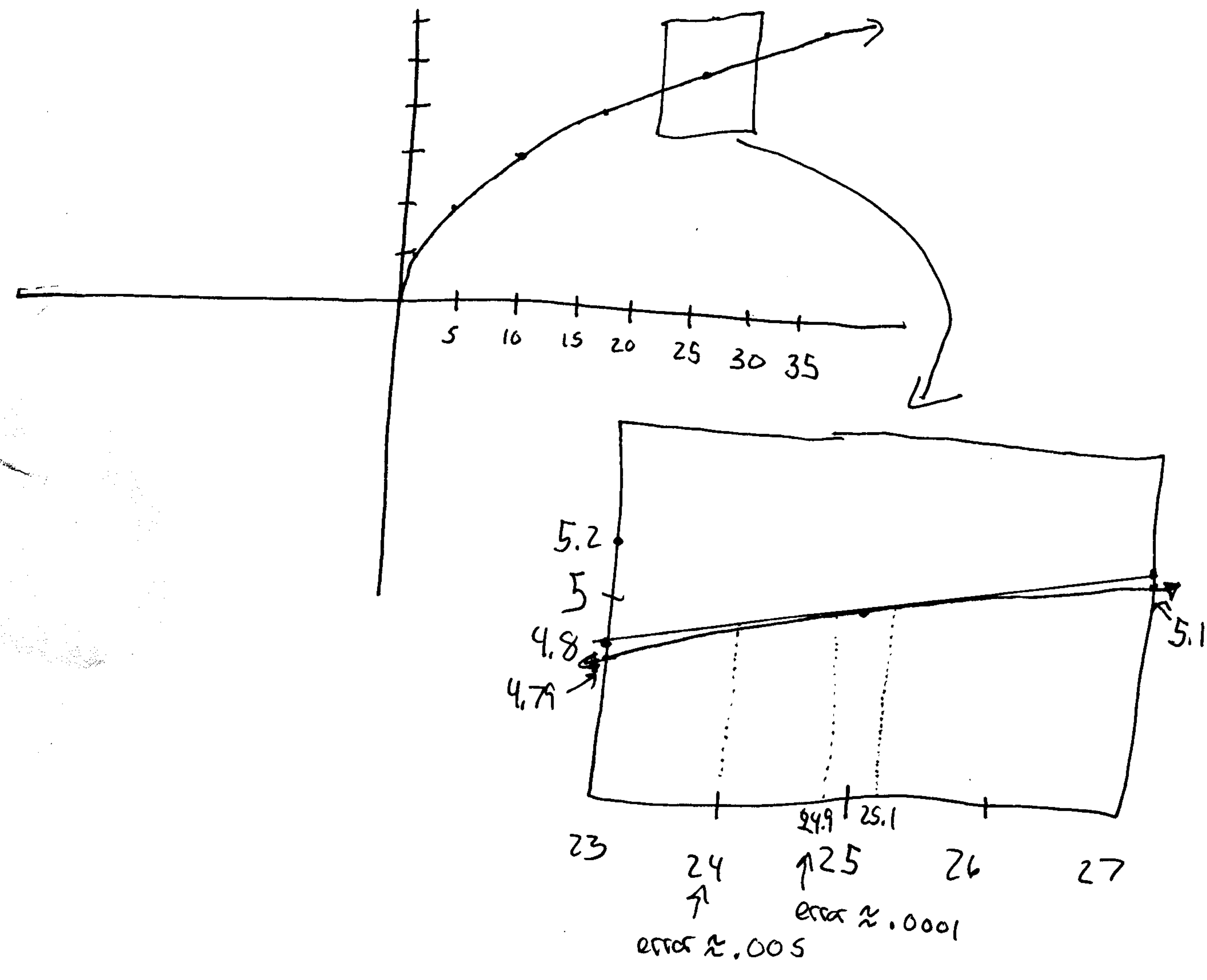
a) Here $x = 23$, so using the linear approximation, slope $= \frac{1}{10} = \frac{\text{rise}}{\text{run}} = \frac{\Delta f(x)}{25-23}$

So $\frac{1}{10} = \frac{\Delta f(x)}{2}$, so we estimate $\Delta f(x) = \frac{1}{5}$, so $\boxed{\sqrt{23} \approx 4.8}$

b) $\frac{1}{10} = \frac{\Delta f(x)}{25-24} \Rightarrow \Delta f(x) = \frac{1}{10}$, so $\boxed{\sqrt{24} \approx 4.9}$

c) $\frac{1}{10} = \frac{\Delta f(x)}{25-24.9} \Rightarrow \Delta f(x) = \frac{.1}{10} = \frac{1}{100}$, so $\boxed{\sqrt{24.9} \approx 4.99}$

d) $\frac{1}{10} = \frac{\Delta f(x)}{25-25.1} \Rightarrow \Delta f(x) = \frac{-.1}{10} = \frac{-1}{100}$, so $\boxed{\sqrt{25.1} \approx 5.01}$



2. Here we just the closest integer which is a perfect square:

a) $x = 100$

b) $x = 9$

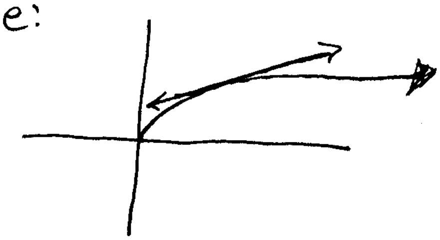
c) $x = 16$

d) $x = 121$

5. $\frac{d}{dx}(x^{1/3}) = \frac{1}{3}x^{-2/3}$, approximate $\sqrt[3]{30}$. First, we want a "nice" value near 30 that we can compute $\sqrt[3]{\ }$ for. Choose $x = 27$. So $f(x) = x^{1/3}$, $f'(x) = \frac{1}{3}x^{-2/3}$, so our tangent line at $27 = f'(27) = \frac{1}{3}\frac{1}{(\sqrt[3]{27})^2} = \frac{1}{3}\cdot\frac{1}{9} = \frac{1}{27}$

So now, $\frac{1}{10} = \frac{\text{rise}}{\text{run}} = \frac{\Delta f(x)}{30-27} \Rightarrow \Delta f(x) = \frac{3}{10}$. So $f(30) \approx f(27) - \Delta f$

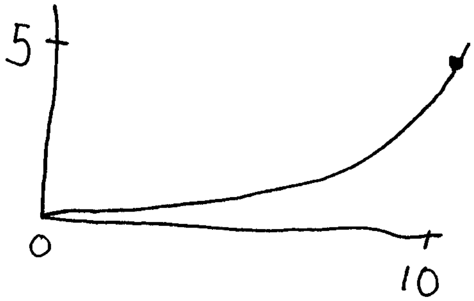
$f(30) \approx 26.7$. We know this is a bit too high, because when we draw a tangent line:



the function curves away at both ends, so the true value is always below the value we extrapolate along the line.

6. a) The steamboat is still in St. Paul at $t = 0$, so $s(0) = 0$ (we can specify it numerically). The boat is moving a 10mph, so $s(x)$ is getting bigger at 0, and $s'(0) = 10$. We also know it is accelerating, so s' is getting bigger at 0, so $s''(0)$ is also positive..

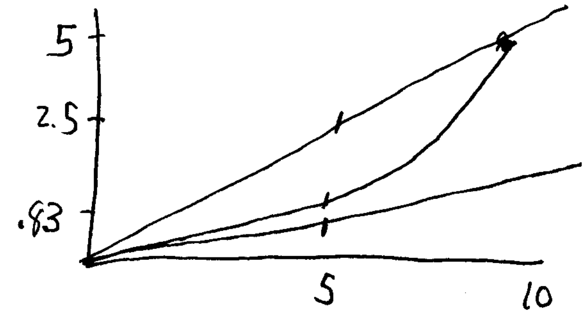
b)



Since $s'(0)$ is positive, we know how to start drawing the graph. Since $s''(0)$ is positive, we know it's concave up.

c) Since the boat is constantly accelerating, it will go faster for the second half than the first half, so it can't have gotten more than halfway there in half the time. Also, since the boat started at 10 mph and is only getting faster, it must have gone at least $10 \frac{m}{hour} \cdot \frac{5}{60} \text{hours} = .8$

So $\boxed{.83 < s(5) < 2.5}$



2. $V(r) = \frac{4}{3}\pi r^3$, the function for the volume of a sphere.

Let's plug in a number for r like 100: $V(r) \approx 4{,}186{,}666$
Now lets try $r = 101$: $V(r) \approx 4{,}313{,}526$
So it grew by 126,860.

The surface area: $S(r) = 4\pi r^2$, which, evaluated at 100, is 125,600. Pretty close. Let's do a limit: $\lim_{h\to 0}\left[\frac{V(r+h)-V(r)}{h}\right] =$

$$\lim_{h\to 0}\left[\frac{\frac{4}{3}\pi(r+h)^3 - \frac{4}{3}\pi(r)^3}{h}\right] = \lim_{h\to 0}\left[\frac{\frac{4}{3}\pi(r^3+3r^2h+3rh^2+h^3) - \frac{4}{3}\pi r^3}{h}\right]$$

$$= \lim_{h\to 0}\left[(3r^2+3rh+h^2)\cdot\frac{4}{3}\pi\right] = 4\pi r^2$$, which is $S(r)$. No coincidence.

3. a) $D(t)$ is positive, because the diameter of roll has to be positive
b) $D'(t)$ is positive because the roll is getting bigger.
c) $D''(t)$ is negative, because it starts taking more & more sheets to make it all the way around the roll & so the time to wrap around once and add a set amount to the diameter increases.