Sol'n Set 17

8.3 / 2. $f(x) = \frac{x-2x^2}{5} = \frac{1}{5}x - \frac{2}{5}x^2$, so

$f'(x) = \frac{1}{5}(1)x^0 - \frac{2}{5}(2)x^1 = \boxed{\frac{1}{5} - \frac{4}{5}x}$

3. $f(x) = \pi(3x^2+7x+1)(x-2) = (3x^2+7x+1)(\pi x - 2\pi)$

$f'(x) =$ first * derivative of second + second * derivative of first

$= (3x^2+7x+1)(\pi(1)x^0 - 0) + (\pi x - 2\pi)(3(2)x^1 + 7(1)x^0 + 0)$

$= 3\pi x^2 + 7\pi x + \pi + 6\pi x^2 + 7\pi x - 12\pi x - 14\pi$

$= \boxed{9\pi x^2 + 2\pi x - 13\pi}$

8. $f(x) = \frac{a}{x} + bx(c-dx) = \frac{a}{x} + bcx - bdx^2 = ax^{-1} + bcx - bdx^2$

$f'(x) = a(-1)x^{-2} + bc(1)x^0 - bd(2)x^1 = \boxed{\frac{-a}{x^2} + bc - 2bdx}$

9. a) $f(x) = \frac{(x^2+2x)x}{3} = \frac{x^3+2x^2}{3} = \boxed{\frac{1}{3}x^3 + \frac{2}{3}x^2}$

b) $f(x) = \frac{(x^2+1)^2}{x} = \frac{x^4+2x^2+1}{x} = \frac{x^4}{x} + \frac{2x}{x} + \frac{1}{x} = \boxed{x^3 + 2 + x^{-1}}$

11. a) $f(x) = (x+1)(x-1)x = (x^2-1)x = \boxed{x^3 - x}$

b) $f(x) = \frac{x + \frac{1}{x}}{x} = \frac{x}{x} + \frac{\frac{1}{x}}{x} = 1 + \frac{1}{x^2} = \boxed{1 + x^{-2}}$

17. $f(x) = \begin{cases} x^3 & x \le 1 \\ kx & x > 1 \end{cases}$

a) f continuous means x^3 should touch kx at $x=1 \Rightarrow 1^3 = k \cdot 1 \Rightarrow \boxed{k=1}$

b) for f to be differentiable, the left & right-hand limits must be the same. Along the left, $f(x) = x^3$, along the right: $f(x) = x$

So for f to be differentiable, $f'(1)$ must be the same: $\frac{d}{dx}x^3 @ 1 = 3x^2 @ 1 = 3$

$\frac{d}{dx} x @ 1 = 1 @ 1 = 1$. $3 \neq 1$, so f is <u>not differentiable</u> at 1.

18. $f(x) = x(x^2+2) = x^3+2x$, so $f'(x) = 3x^2+2$.

The tangent line at $x=1$ has slope = the rate of change of $f(x)$ at $x=1$ $\Rightarrow$ the slope $= f'(1) = 5$. Since the tangent line goes through the point $x=1$, $f(x)=3$, we get:

$y = mx + b$, $m = 5$

$3 = 5\cdot(1) + b$

$b = -2$

so $\boxed{y = 5x - 2}$