Soln Set 18

8.3/20. $\frac{d}{dx}\ \frac{x+1}{x^3+3x+1}$. Say $f(x)=x+1$, $g(x)=x^3+3x+1$

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)\cdot g(x) - g'(x)f(x)}{[g(x)]^2}$$

$$= \frac{1\cdot(x^3+3x+1) - (3x^2+3)(x+1)}{(x^3+3x+1)^2}$$

$$= \frac{x^3+3x+1 - (3x^3+3x^2+3x+3)}{(x^3+3x+1)^2}$$

$$= \boxed{\frac{-2x^3-3x^2-2}{(x^3+3x+1)^2}}$$

21. $\frac{d}{dx}\left(\frac{\pi}{\pi x+\pi}\right) = \frac{d}{dx}\left(\frac{1}{x+1}\right)$. Say $f(x)=1$, $g(x)=x+1$

then $\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$

$$= \frac{0(x+1) - 1(1)}{[x+1]^2} = \boxed{\frac{-1}{(x^2+2x+1)}}$$

22. $\frac{d}{dx}\left(\frac{2x^2+x+1}{\sqrt{2x}}\right)$. Say $f(x)=2x^2+x+1$, $g(x)=\sqrt{2x}=\sqrt{2}\cdot x^{1/2}$

$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$ $\qquad g'(x) = \sqrt{2}\left(\frac{1}{2}\right)x^{-1/2}$

$$= \frac{(4x+1)(\sqrt{2}x^{1/2}) - \left(\frac{\sqrt{2}}{2}x^{-1/2}\right)(2x^2+x+1)}{(\sqrt{2x})^2}$$

$$= \frac{(4x+1)(\sqrt{2x}) - \frac{1}{\sqrt{2x}}(2x^2+x+1)}{2x}$$

$$= \boxed{\frac{1}{2x}(4x+1) - \frac{1}{\sqrt{8x^3}}(2x^2+x+1)}$$

9.1/ 1.a)

days	population
0	200
1	600
2	1800
3	5400
...	...

$\boxed{P(t) = 3^t \cdot 200}$

b) $t=-1$, $P(-1) = 3^{-1} \cdot 200 \boxed{\approx 66}$. $t=-4 \Rightarrow P(-4) = 3^{-4} \cdot 200 \boxed{\approx 2}$

c) $\boxed{P(t) = B_0 \cdot 3^t}$

d) w = # of weeks. $7w$ = 1 day = t

So $\boxed{P(w) = B_0 3^{7w}}$

e) $P(w)$ @ $w=1 \Rightarrow P(1) = B_0 \cdot 3^{7 \cdot 1} = \boxed{2187 B_0}$

2.a) 2^{64}, which is on the order of 10^{19}.

b) $(.02 \frac{g}{grain}) \cdot (\frac{1}{1000} \frac{kg}{g}) \cdot (\frac{1}{907.18} \frac{tons}{kg})(1.84 \times 10^{19} grains) \approx 4.07 \times 10^{11}$ tons

which is about 1000 times the annual world production.

c)

Sum of powers of 2	total
1	1
1+2	3
1+2+4	7
1+2+4+8	15
1+2+4+8+16	31
...	...
$1+2+\ldots+2^n$	$2^{n+1}-1$

So we have $(.02 \frac{g}{grain})(\frac{1}{1000} \frac{kg}{g})(\frac{1}{907.18} \frac{tons}{kg})(2^{64}-1 \text{ grains})$

$\approx \boxed{8.14 \times 10^{11} \text{ tons}}$