

9.3/ 1.a)

$$C(t) = C_0 \cdot 2^{-\frac{t}{5730}}$$

We can get this by knowing we

must be dealing with an exponential equation. We have half-lives, which implies a power of 2. The population gets smaller as time passes, so we have a negative power. When $t=5730$, we have $\frac{1}{2}$ the initial population, so $\frac{t}{5730}$ in the exponent, and when $t=0$, the population should $= C_0$, so C_0 must be the coefficient out front.

b) $t = 2000 - 137 = 1863$ years. $C_0 =$ percentage of original C_{14} at $t=0$, which is 100%. $C(1863) = 100\% \cdot 2^{-\frac{1863}{5730}} = \boxed{79.8\%}$

c) $C(t) = C_0 \cdot \left(2^{-\frac{1}{5730}}\right)^t$, which means each year, the current percentage is multiplied by $2^{-\frac{1}{5730}} \approx 99.988\%$. That means the population decreases by $1 - 99.988\% = \boxed{.012\%}$

4. a) If it is linear, then the population increases by 100,000 people every 20 years, so it would increase by 50,000 in 10 years, so the population would be equal to 150,000 in 1980.

b) If increasing exponentially, it starts off growing more slowly than it ends, so most of the population increase comes at the end, so in 10 years, we should be less than halfway there, so the population should be less than 150,000.

5. a) $D(t)$ increases linearly with time. $H(t)$ decreases exponentially with time, and $J(t)$ increases exponentially with time.

b) You could add them all up and find that Henry is better off, or just notice that he starts at 50,000 (lots bigger than anybody's) and ends at ≈ 45600 , just slightly smaller than where others end up.

c) If you have a future, you want an exponentially increasing salary (bills).

13. a) $P(t) = (1 - .02)^t = \boxed{.98^t}$

b) $\frac{1}{2} = (.98)^t$, so $t \approx \boxed{34 \text{ years}}$

9.4/ 5. $\frac{d}{dx}(x^3 e^x) = x^3 \frac{d}{dx} e^x + e^x \frac{d}{dx} x^3$ by Product Rule.
 $= x^3 e^x + e^x \cdot 3x^2 = \boxed{(x^3 + 3x^2)e^x}$

9. $\frac{d}{dx}(e^{2x}(x^2+2x+2)) = \left[\frac{d}{dx}(e^{2x})\right](x^2+2x+2) + e^{2x}\left[\frac{d}{dx}(x^2+2x+2)\right]$ by Product Rule.

Note $e^{2x} = e^x \cdot e^x$, so $\frac{d}{dx}(e^{2x}) = \frac{d}{dx}(e^x \cdot e^x) = e^x \frac{d}{dx} e^x + e^x \frac{d}{dx} e^x = 2e^{2x}$

So an equation above becomes $[2e^{2x}](x^2+2x+2) + e^{2x}(2x+2)$
 $= e^{2x}(2x^2+4x+4) + e^{2x}(2x+2) = \boxed{e^{2x}(2x^2+6x+6)}$

14. $M(t) = C_0 b^t$ is just the general form of an exponential equation.

$M'(t) = C_0 M(t) \ln b$ is the derivative, as they show in the book

Presumably, $C_0 =$ the amount of money in the bank $= 4000$ in the first case

then $M'(0) = 100 = 4000 \cdot M(0) \Rightarrow M(0) = \frac{1}{40}$.

For $C_0 = 5500$, $M'(0) = 5500 \cdot M(0) = 5500 \cdot \frac{1}{40} = \boxed{137.5}$