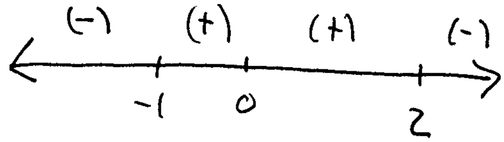
10.1\ 5. Critical points are where $f'(x_0)=0$, $f'(x_0)=$ undefined, or x_0 is an endpoint of the domain. Our domain is $-\infty$ to ∞, so we have no endpoints.

Sol'n Set 21

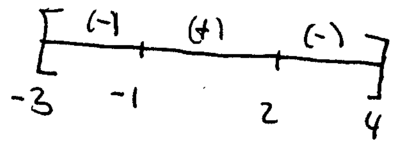
$f(x)=-2x^3+3x^2+12x+5$, so $f'(x)=-6x^2+6x+12$, which is never undefined, but it $=0$ when $\boxed{x=2 \text{ or } -1}$

$f'(x)$ is:



So 2 is a local max & -1 is a local min.
These aren't absolute extrema because $f(x)$ increases & decreases without bound on $-\infty$ to ∞

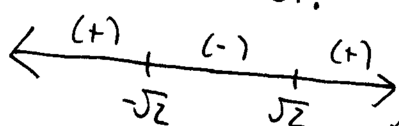
6. $f(x)=-2x^3+3x^2+12x+5$, on $[-3,4]$. This is the same as #5, only now we have endpoints at $x=-3$, $x=4$.



, So -3 is a local maximum
-1 is a local minimum
2 is a local maximum
4 is a local minimum.

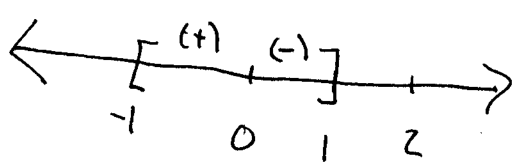
And plugging these into $f(x)$, we find the absolute max is at -3, where $f(x)=50$
absolute minimum is at 4, where $f(x)=-27$

7. $f(x)=x^5-20x+5$ on $(-\infty,\infty)$. No endpoints. $f'(x)=5x^4-20$ (which is never undefined). $f'(x)=0$ when $\boxed{x=\pm\sqrt{2}}$ (throw out imaginary solutions because we are on the real line here).



So $-\sqrt{2}$ is a local max, $\sqrt{2}$ is a local min
These are not absolute because $f(x)$ increases & decreases without bound on $-\infty$ to ∞.

11. $f(x)=3x^4-8x^3+3$ on $[-1,1]$. Endpoints at $\boxed{x=\pm 1.}$
$f'(x)=12x^3-24x^2=0$ when $\boxed{x=0 \text{ or } \cancel{x=2}.}$ $x=2$ is thrown out because it isn't on our interval.



-1 is a relative (local) min
0 is a local max
1 is a local min

And evaluating these in $f(x)$, we find 0 is an absolute max, -1 is an absolute min.

17. $f(x) = xe^x$

a) $f'(x) = x\frac{d}{dx}e^x + e^x\frac{d}{dx}x = xe^x + e^x = (x+1)e^x = 0$ when $\boxed{x = -1}$

b) $(-)$ | $(+)$ at -1 So -1 is a local minimum

c) $f(x)$ increases without bound as $x \to \infty$, so no absolute max.

d) -1 is an absolute minimum, because $f(x)$ always increases to either side of it. $\boxed{f(-1) = \frac{-1}{e}}$

e) if x is restricted to $(-\infty, 0]$, we get a new point (0) to check things for. $f(0) = 0$, and $f(x)$ is negative over $(-\infty, 0]$, so 0 is an absolute max. 0 doesn't affect the calculated minimum.

20. $f(x) = \frac{10x}{x^2+1}$

$$f'(x) = \frac{\left(\frac{d}{dx}(10x)\right)(x^2+1) - 10x\left(\frac{d}{dx}(x^2+1)\right)}{(x^2+1)^2} = \frac{10x^2+10-20x^2}{(x^2+1)^2} = \frac{-10x^2+10}{(x^2+1)^2}$$

This is undefined at $x = \sqrt{-1}$, but this is not a real number, so no worries.

$f'(x) = 0$ when $\boxed{x = \pm 1}$

$f'(x)$: $(-)$ | $(+)$ | $(-)$ at -1, 1 so -1 is a local min, 1 is a local max.

$f(1) = 5$, $f(-1) = -5$. Let's see if there are any other places where $f(x) = 5$ or -5:

$$5 = \frac{10x}{x^2+1} \Rightarrow 5x^2 + 5 = 10x \Rightarrow x^2 + 1 = 2x \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow x = 1$$

and similarly $f(x) = -5$ only at -5. So this means the function $f(x)$ never gets above 5 or -5, so -1 and 1 are absolute maxima & minima.