Soln Set 30

14.1/1. $\frac{d}{dx}(2\ln(5x)) = \frac{d}{dx}(2(\ln 5 + \ln x))$

$= \frac{d}{dx}(2\ln 5 + 2\ln x) = 0 + 2\cdot\frac{1}{x} = \boxed{\frac{2}{x}}$

2. $\frac{d}{dx}(\pi \ln\sqrt{x}) = \frac{d}{dx}(\frac{\pi}{2}\ln x) = \frac{\pi}{2}\cdot\frac{1}{x} = \boxed{\frac{\pi}{2x}}$

7. $\frac{d}{dx}(\frac{\log_2 x}{3}) = \frac{d}{dx}(\frac{1}{3}\log_2 x) = \frac{1}{3}\frac{1}{\ln 2}\cdot\frac{1}{x} = \boxed{\frac{1}{3\ln 2}\cdot\frac{1}{x}}$

8. $f(x) = \frac{\ln\sqrt{3x}}{2} + 3 = \frac{1}{4}\ln(3x) + 3 = \frac{1}{4}\ln x + \frac{1}{4}\ln 3 + 3$

We know $\ln x$ is 1-to-1 because its derivative is $\frac{1}{x}$, which is positive over the range for which $\ln x$ is defined. We also know $\ln x$ is invertible because it's defined to be the inverse of $e^x$. Since $\ln(x)$ is invertible, $\ln(x) + C$ must be invertible, so $f(x)$ is invertible.

$x = \frac{1}{4}\ln(f^{-1}(x)) + \frac{1}{4}\ln 3 + 3 \Rightarrow \ln(f^{-1}(x)) = x - \ln 3 - 12$

$\Rightarrow f^{-1}(x) = e^{(x-\ln 3-12)} = e^x\cdot e^{-\ln 3}\cdot e^{-12} = \boxed{\frac{e^x}{3e^{12}}}$

10. $f(x) = x\ln(x) \Rightarrow f'(x) = x\cdot\frac{1}{x} + 1\cdot\frac{1}{x} = \frac{1}{x} + 1$

critical points are where $f'(x) = 0$ or undefined. Our domain for $f(x)$ is $(0,\infty)$ because $\ln(x)$ is not defined for $x \le 0$. So $f(x)$ isn't 0 anywhere. Our domain has its endpoints at $0$ & $\infty$. $f(\infty) = \infty \Rightarrow \infty$ is an absolute maximum. $\lim_{x\to 0^+} f(x) = -\infty$. However $0$ is not in the domain of $f(x)$, so we can't call it a minimum.

14. a) our domain is $x \in (0,\infty)$.

b) critical points where $f'(x) = 0$ or undefined, or at endpoints. $f'(x) = \frac{1}{x} - 1$.
$f'(x) = 0$ when $x = 1$. $f'(x)$ is undefined at $x = 0$, but $0$ is not in our domain. Our only endpoint that's in our domain is $\infty$. So our points are $x = 1, \infty$.
$f'(x)$ is negative for $x > 1$, positive $< 1$, so $x = 1$ is a local maximum. Since $f'(x)$ is negative after 1, $\infty$ must be a local minimum. So our max is at $(1,-1)$. Our local min is $(\infty,-\infty)$.

14 c) $f'' = -\frac{1}{x^2}$. $f''$ is negative over the entire domain.

$\Rightarrow f$ is concave down.

d)