Soln Set 31

4. $f(x) = x \cdot 2^x$ has critical points where

$f'(x) = 2^x + x \ln 2 \cdot 2^x = 0 \Rightarrow x \ln 2 = -1 \Rightarrow \boxed{x = \frac{-1}{\ln 2}}$

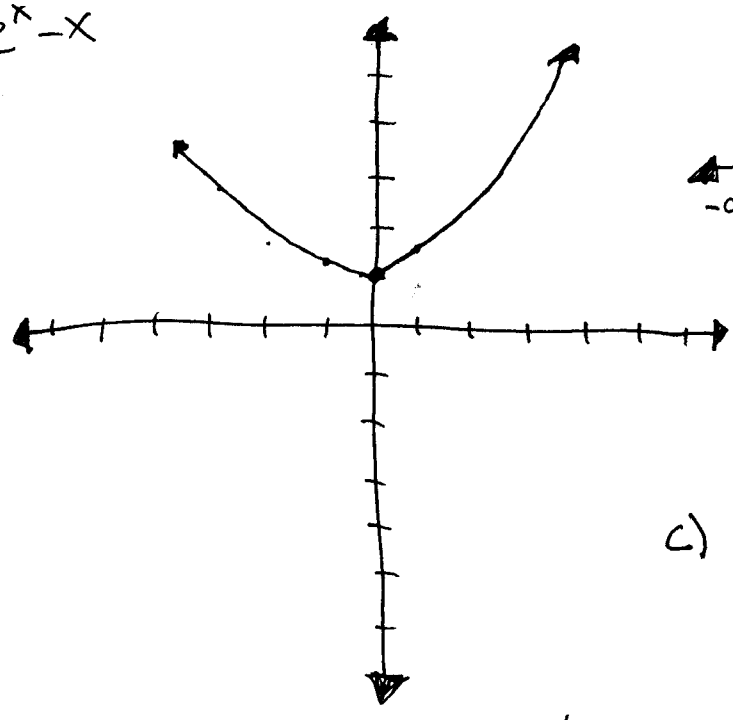
5. a) $f(x) = x^2 + e^x + x^e + e^2 \Rightarrow f'(x) = 2x + e^x + e x^{(e-1)} + 0$

$= \boxed{2x + e^x + e x^{(e-1)}}$

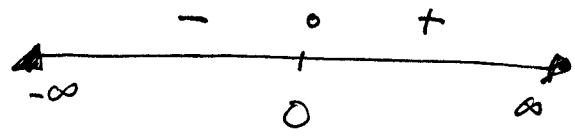
b) $f(x) = \pi e^x - \frac{6e^x}{\sqrt{29}} \Rightarrow f'(x) = \boxed{\pi e^x - \frac{6}{\sqrt{29}} \cdot e^x}$

c) $f(x) = 3e^{x+3} \Rightarrow f'(x) = \frac{d}{dx}(3e^{x+3}) = \frac{d}{dx}(3e^3 e^x) = 3e^3 e^x = \boxed{3e^{x+3}}$

6. $f(x) = e^x - x$



$f'(x) = e^x - 1$



extremum (abs. minimum) at $(0, 1)$.

c) $f(x) = 5$ has 2 solutions.

$f(x) = .5$ has no solutions.

143/ 1. $f(x) = \frac{3 \ln \sqrt{x}}{x} \Rightarrow f'(x) = 3\left[\frac{\frac{d}{dx}(\ln x^{1/2}) \cdot x - \ln x^{1/2} \cdot 1}{x^2}\right] = \frac{\frac{1}{2} \cdot \frac{1}{x} \cdot x - \frac{1}{2} \cdot \frac{1}{x}}{x^2}$

$= \frac{1}{2x^2} - \frac{1}{2x^3}$. $\boxed{f'(e) = \frac{1}{2e^2} - \frac{1}{2e^3}}$

6. $f(x) = \ln(3x^5) = \ln 3 + \ln x^5 = \ln 3 + 5 \ln x$

$f'(x) = 0 + 5\left(\frac{1}{x}\right) = \boxed{\frac{5}{x}}$

11. $f(x) = \frac{x + \ln \frac{1}{x}}{x^2} = \frac{1}{x} + \frac{1}{x}(\ln x^{-1}) = \frac{1}{x} + \frac{-1}{x} \ln x$

$f'(x) = -x^{-2} + \left(\frac{-1}{x} \cdot \frac{1}{x} + \frac{1}{x^2} \cdot \ln x\right) = \frac{-2}{x^2} + \frac{\ln x}{x^2} = \boxed{\frac{\ln x - 2}{x^2}}$

24. a) $P(t) = P_0 e^{kt}$. $P(0) = P_0 = 200$, $P(30) = 900 = P_0 e^{30k}$

$900 = 200e^{30k} \Rightarrow e^{30k} = \frac{9}{2} \Rightarrow 30k = \ln\frac{9}{2}$, $k = \boxed{\frac{1}{30}\ln\frac{9}{2}}$

b) $\boxed{M(t) = 100 \cdot 2^{\frac{t}{10}}}$

c) $M(t) = P(t) \Rightarrow 100 \cdot 2^{\frac{t}{10}} = 200\, e^{(\frac{1}{30}\ln\frac{9}{2})t} \Rightarrow \frac{1}{2} \cdot 2^{\frac{t}{10}} = e^{\frac{1}{30}(\ln\frac{9}{2})t}$

$\Rightarrow 2^{\frac{t}{10}-1} = e^{\frac{1}{30}(\ln\frac{9}{2})t} \Rightarrow (\frac{t}{10}-1)\ln 2 = \frac{1}{30}(\ln\frac{9}{2})t \ln e$

$\Rightarrow \frac{t}{10} - 1 = \frac{1}{30}\frac{\ln\frac{9}{2}}{\ln 2}t \Rightarrow \left[\frac{1}{30}\frac{\ln\frac{9}{2}}{\ln 2} - \frac{1}{10}\right]t = -1$

$t = \dfrac{-1}{\frac{1}{30}\frac{\ln\frac{9}{2}}{\ln 2} - \frac{1}{10}} = \dfrac{30}{3 - \log_2\frac{9}{2}} = \dfrac{30}{3-[\log_2 9 - \log_2 2]} = \boxed{\dfrac{30}{4-\log_2}}$

d) $P'(t) = \frac{d}{dt}(P_0 e^{kt}) = P_0 \frac{d}{dt}((e^k)^t) = P_0 (\ln e^k)(e^k)^t = P_0 \cdot k e^{kt}$

$= 200 \cdot \frac{1}{30}\ln\frac{9}{2} \cdot e^{\frac{1}{30}\ln\frac{9}{2}t} = \boxed{\frac{20}{3}\ln\frac{9}{2}\, e^{\frac{1}{30}\ln\frac{9}{2}t}}$

e) $M'(t) = \frac{d}{dt}(100 \cdot 2^{t/10}) = 100 \cdot \frac{d}{dt}((2^{1/10})^t) = 100 \ln 2^{1/10} (2^{1/10})^t$

$= 100 \cdot \frac{1}{10}\ln 2 \cdot 2^{t/10} = \boxed{10\ln 2 \cdot 2^{\frac{t}{10}}}$

f) $P'\left(\frac{30}{4-\log_2 9}\right) = \frac{20}{3}\ln\frac{9}{2}\, e^{\ln\frac{9}{2} \cdot \frac{30}{4-\log_2 9}} \approx 61.4$

$M'\left(\frac{30}{4-\log_2 9}\right) = 10\ln 2 \cdot 2^{\frac{3}{4-\log_2 9}} \approx 84.9$

So mosquitos are growing faster.