Sol'n Set 33

16.3/ 2) a) i) $(\ln x)(\ln x) = (\ln x)^2$ ✓

ii) $\ln x^2 \neq (\ln x)^2$ (False)

iii) $\ln[(x)(x)] = \ln x^2 \neq (\ln x)^2$ (False)

iv) $2\ln x = \ln x^2 \neq (\ln x)^2$ (False)

b) i) $(\ln x)(\ln x) = (\ln x)^2 \neq 2\ln x$ (False)

ii) $\ln x^2 = 2\ln x$ ✓

iii) $\ln[(x)(x)] = \ln x^2 = 2\ln x$ ✓

c) $\frac{dy}{dx} = \frac{d}{dx}(\ln x)^2 = 2(\ln x)^1 \cdot \frac{1}{x} = \boxed{\frac{2}{x}\ln x}$

$= \frac{d}{dx}[(\ln x)(\ln x)] = \frac{1}{x}\ln x + \ln x\frac{1}{x} = \boxed{\frac{2}{x}\ln x}$

d) $\frac{dy}{dx} = \frac{d}{dx}(\ln x^2) = \frac{d}{dx}(2\ln x) = \boxed{\frac{2}{x}}$

$= \frac{d}{dx}(\ln x^2) = \frac{1}{x^2} \cdot 2x = \boxed{\frac{2}{x}}$

3) a) a) $e^{-(x)(x)} = e^{-x^2}$ ✓

b) $(e^{-x})^x = e^{-x \cdot x} = e^{-x^2}$ ✓

c) $\left(\frac{1}{e^x}\right)^x = (e^{-x})^x = e^{-x^2}$ ✓

d) $\left(\frac{1}{e^x}\right)^2 = (e^{-x})^2 = \underline{\underline{e^{-2x} \neq e^{-x^2}}}$

e) $(e^{-x})^2 = \underline{\underline{e^{-2x} \neq e^{-x^2}}}$

f) $(e^{-2})^x = \underline{\underline{e^{-2x} \neq e^{-x^2}}}$

g) $\underline{\underline{e^{-2x} \neq e^{-x^2}}}$

h) $e^{(-x)^2} = \underline{\underline{e^{x^2} \neq e^{-x^2}}}$

4. $f(x) = e^{-x^2}$

a) domain: all x, range: $(0, 1]$

b) f is even because it's symmetric for $\pm x$ (across y axis)

c) f increasing for $x<0$, f decreasing for $x>0$

d) relative maximum where $f'=0 \Rightarrow \underline{x=0}$. It's an absolute ma

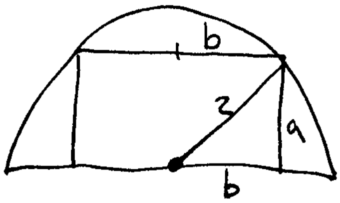
e) f has an absolute maximum at $x=0$, $f(x)=1$. There is no absolute minimum value, but there is a lower bound at $f(x)=0$.

f) $f''(x) = \frac{d}{dx}\left(e^{-x^2}\cdot(-2x)\right) = -2e^{-x^2} + (-2x)\left(e^{-x^2}(-2x)\right)$

$$= -2e^{-x^2} + (4x^2)e^{-x^2} = (4x^2-2)e^{-x^2}$$

$=0$ when $x^2 = \frac{1}{2}$, $\boxed{x = \pm\sqrt{\frac{1}{2}}}$

5.



$\Rightarrow a^2+b^2=2^2$. Want to maximize $a\cdot(2b)$.

$b = \sqrt{4-a^2}$

$V = a(2b) = a\cdot 2\sqrt{4-a^2}$

$= \sqrt{16a^2-4a^4}$

$$\frac{dV}{da} = \frac{d}{da}\left[(16a^2-4a^4)^{\frac{1}{2}}\right] = (16a^2-4a^4)^{-\frac{1}{2}}\cdot(32a-16a^3) = 0$$

$\Rightarrow$ either $16a^2-4a^4=0$ or $32a-16a^3=0$

$\Rightarrow a=0$ or ± 2 or $a=0$, $a=\pm\sqrt{2}$

at $a=0, 2$ $V=0$, so these aren't our maxima.

we rule out any $a<0$, so our answer is $\boxed{\sqrt{2}}$

This gives us $2b = 2\sqrt{2}$, and $\boxed{V=4}$

7 a) $$\frac{dy}{dx}=\frac{d}{dx}\left[\frac{1}{x\ln 2+1}\right]=\frac{-1}{(x\ln 2+1)^2}\cdot(\ln 2)=\boxed{\frac{-\ln 2}{(x\ln 2+1)^2}}$$

b) $$\frac{dy}{dx}=\frac{d}{dx}\left[\ln(5x^3+8x)\right]=\frac{1}{5x^3+8x}\left(15x^2+8\right)=\boxed{\frac{15x^2+8}{5x^3+8x}}$$

c) $$\frac{dy}{dx}=\frac{d}{dx}\left[2^x(x^2+x)^7\right]=2^x\cdot\frac{d}{dx}\left[(x^2+x)^7\right]+\ln 2\cdot 2^x(x^2+x)^7$$

$$=2^x\cdot 7(x^2+x)^6\cdot(2x+1)+2^x\cdot\ln 2\,(x^2+x)^7$$

$$=(x^2+x)^6\cdot 2^x\left[14x+7+(x^2+x)\ln 2\right]$$

$$=\boxed{(x^2+x)^6\cdot 2^x\left[(\ln 2)x^2+(14+\ln 2)x+7\right]}$$

d) $$\frac{dy}{dx}=\frac{d}{dx}\left[(\ln(5x)+e^{6x})^{1/2}\right]=(\ln(5x)+e^{6x})^{-1/2}\cdot\left[\frac{1}{x}\cdot 5+e^{6x}\cdot 6\right]$$

$$=\boxed{\frac{\frac{5}{x}+6e^{6x}}{\sqrt{\ln(5x)+e^{6x}}}}$$

e) $$y=\frac{7}{\sqrt{\ln x}}\Rightarrow\frac{dy}{dx}=\frac{d}{dx}\left[7\cdot(\ln x)^{-1/2}\right]=\frac{-7}{2}(\ln x)^{-3/2}\cdot\frac{1}{x}$$

$$=\boxed{\frac{-7}{2x\ln x\sqrt{\ln x}}}$$

f) $$y=4^{x/3}\cdot\ln 3x\Rightarrow\frac{dy}{dx}=\frac{d}{dx}\left[4^{x/3}\cdot\ln 3x\right]=(\ln 4)4^{x/3}\cdot\frac{1}{3}\cdot\ln 3x+4^{x/3}\cdot\frac{1}{3x}\cdot 3$$

$$=\boxed{4^{x/3}(\ln 4)(\ln 3x)+4^{x/3}\cdot\frac{1}{x}}$$