# Practice Problems: Test #1, Set #2

Important Information:

1.   The first test will be held on **Wednesday October 30 from 7-9pm in Science Center D**.

2.   The test will include approximately eight problems (each with multiple parts).

3.   You will have 2 hours to complete the test.

4.   You may use your calculator and one page (8" by 11.5") of notes on the test.

5.   The specific topics that will be tested are:

   - The definition of a function.
   - Representations of functions (graphs, numbers/tables, written descriptions, equations).
   - Interpreting graphs, tables, words and symbols.
   - Modeling relationships using linear functions.
   - Interpreting the parameters of a linear function.
   - Calculating and interpreting rates of change.
   - Exponential growth and exponential functions.
   - Modeling relationships using exponential functions.
   - Solving equations involving linear and exponential functions (logarithms).
   - Rates of change and concavity.
   - Approximating functions using their rates of change (Euler's method).
   - Transformations of functions (shifts, stretches and reflections of functions).
   - Power functions.
   - Predicting the appearance of the graph of a power function.
   - Modeling relationships using power functions.
   - Polynomial functions.
   - Predicting the appearance of the graph of a polynomial, exponential or power function.
   - Fitting a polynomial function to data.
   - Compositions of functions.
   - The concept of the inverse of a function.
   - Functions defined in pieces.

6.   The problems included here have been chosen because they are representative of many of the mathematical concepts that we have studied. There is no guarantee that the problems that appear on the test will resemble these problems in any way whatsoever.

7.   Good places to go for help include:

   - Office hours.
   - The labs on Tuesday 10/29.
   - The Math Question Center
   - The course-wide review on Tuesday 10/29 from 5:00-7:00pm in Science Center D.

8.   Remember: On exams, you will have to supply evidence for your conclusions, and explain why your answers are appropriate.

**1.** The table below shows the average annual earnings of full-time, college-educated female workers in 1997[1].

Education	Age=18	Age=25	Age=35	Age=45	Age=55
Some college education	15506	24127	28561	31350	29535
College graduate	26297	37321	46154	45105	40203

(a) Plot a graph showing the average earnings of women who have graduated from college. What kind of polynomial function would do a good job of representing the trends shown in the data?

(b) Find an equation for the average annual earnings of a college graduate as a function of her age.

(c) Express the equation that you have found for the income of college graduates in **vertex form**.

(d) At what age the average female worker (who is a college graduate) achieves her maximum annual earnings and the maximum annual earnings of the average female worker who is a college graduate.

2. In this problem, you are presented with a collection of four polynomial functions. Find formulas for each of them.

(a)

(b)

(c)

(d)
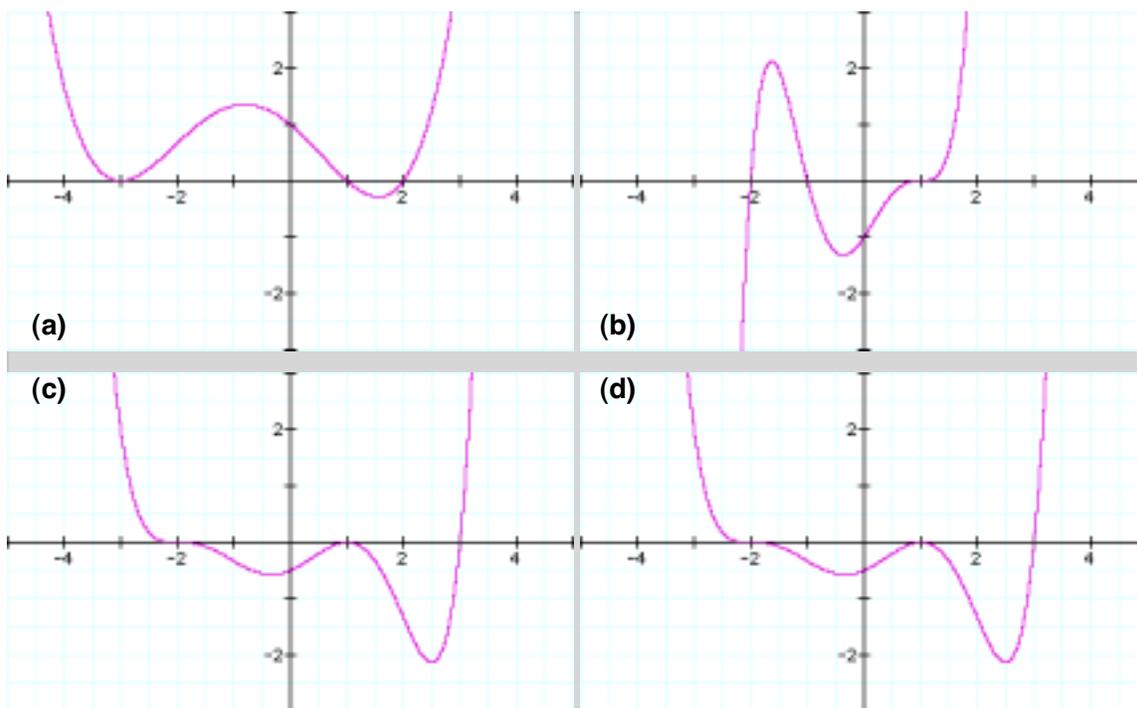
[1] Source: US Bureau of the Census, *Current Population Reports*.

3.   The graph[2] given below shows the share price versus time for the media conglomerate AOL-Time Warner (NYSE: AOL).
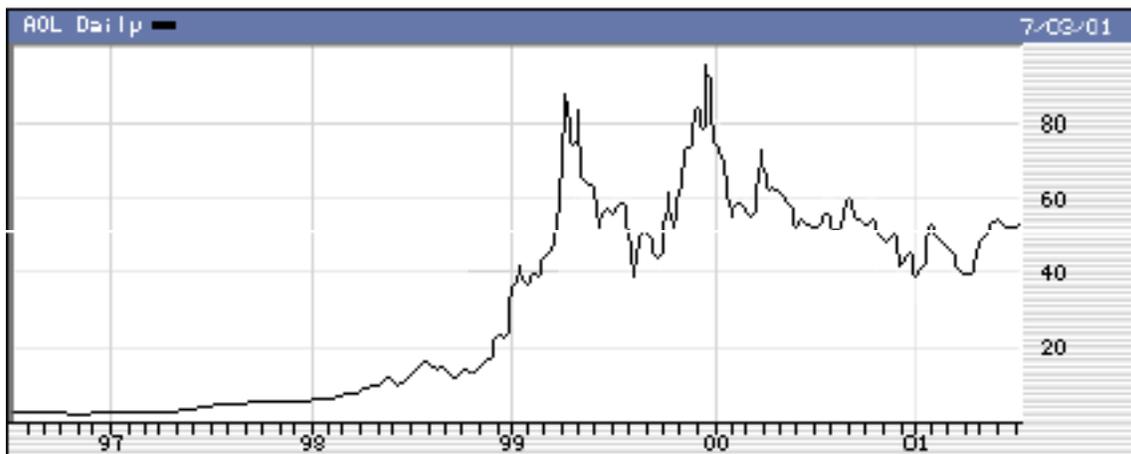
Use this graph to complete Table 1 (shown below). Based on the overall, large-scale shape of this graph, what kind of polynomial function would do a reasonable job of representing share price as a function of time? Use the regression capabilities of your calculator and the values from Table 1 to find an equation for share price as a function of time.

Date	AOL Time Warner Share Price ($)
12/1997	
12/1998	
12/1999	
12/2000	

Table 1.

Table 2 (below) gives the earnings per share (EPS) for AOL-Time Warner[3]. Part (b) makes use of this data.

Year	1998	1999	2000	2001	2002
EPS ($)	0.3	0.48	0.58	1.22	1.57 (estimated)

Table 2: Earnings per Share for AOL-Time Warner, 1998-2002.

(b)   Plot a graph showing the EPS of AOL-Time Warner versus time. Based on your plot, what kind of polynomial function would do a reasonable job of representing the EPS as a function of time? Use the regression capabilities of your calculator and the data in Table 2 to find an equation for EPS as a function of time.

(c)   The price-to-earnings (PE) ratio of a stock is defined to be the price of one share of the stock divided by the earnings per share (EPS). Find an equation that will give the PE ratio of AOL-Time Warner stock as a function of time. What would a graph of PE ratio versus time look like when AOL-Time Warner did not make a profit? (That is, when the earnings per share (EPS) is equal to zero.)

---

4.   Let $f(x)$ and $g(x)$ be the functions defined by the equations given below.

$$f(x) = \sqrt{x} \quad \text{and} \quad g(x) = x^2.$$

(a)   What are the domains and ranges of the functions $f(x)$ and $g(x)$?

(b)   Find an equation for the new function $f(g(x))$. What is the domain and range of this new function?

(c)   Find an equation for the new function $g(f(x))$. What is the domain and range of this new function?

(d)   Are the two new functions $f(g(x))$ and $g(f(x))$ the same? If not, briefly explain why not.

5.   The table shown below shows the number of divorced persons per 1000 married couples in the US from 1970-1998[4].

Year	Number of divorced persons per 1000 married couples.
1970	47
1980	100
1990	142
1995	161
1996	167
1997	177
1998	175

(a)   Plot the number of divorced people per 1000 married couples versus year.

(b)   What kind of function (linear, exponential, power) would do a good job of representing the relationship between number of divorced people and time? Explain your reasoning.

(c)   Find an equation for the function that you have selected that will give number of divorced people (per 1000 married couples) as a function of time.

(d)   No matter which kind of function you have selected, it will have two numbers (called *parameters*). Interpret the meaning of the two parameters of your function in practical terms.

(e)   In practical terms, what is the maximum output from your function? In practice is it possible to reach this maximum value?

6.   In the novel "Catch 22" by Joespeh Heller, the reader is introduced to a character named Major Major. (Major Major is an officer in the army - naturally, his rank is major.) In Chapter 9, Major Major's father is introduced.

> "Major Major's father was a sober God-fearing man whose idea of a good joke was to lie about his age. He was a long-limbed farmer, a God-fearing, freedom-loving, law-abiding rugged individualist who held that federal aid to anybody but farmers was creeping socialism. He advocated thrift and hard work and disapproved of loose women who turned him down. His specialty was alfalfa, and he made a good thing out of not growing any. The government paid him well for every bushel of alfalfa he did not grow. The more alfalfa he did not grow, the more money the government paid him, and he spent every penny he didn't earn on new land to increase the amount of alfalfa he did not produce. Major Major's father worked without rest at not growing alfalfa. On long winter evenings he remained indoors and did not mend harness, and he sprang out of bed at the crack of noon every day just to make certain that the chores would not be done. He invested in land wisely and soon was not growing more alfalfa than any other man in the county. Neighbors sought him out for advice on

[4] Source: US Bureau of the Census, *Marital Status and Living Arrangements* (1999).

all subjects, for he had made much money and was therefore wise. "As we sow, so shall ye reap," he counseled one and all, and everyone said, 'Amen.'"

To get an idea of how quickly exponential functions can grown, we will make the very modest assumptions that Major Major's father starts with an average sized farm for 1910 of 139 acres[5], and that the government pays him enough to increase the amount of land that he has by 1% each year.

(a) Find an equation that will give the size of Major Major's father's farm (in acres) after Major Major's father has been paid not to grow alfalfa for T years.

(b) In 1996, Congress repealed price guarantees on most major crops. If the government payments that Major Major's father depended on were included in this action, how large was the farm when the "jig" was finally up?

(c) How long will it take for Major Major's father's farm to double in size?

(d) The land area of the continental United States is approximately 1,937,678,000 acres. If Major Major's family keeps the same farming situation going, how long will it take them to acquire the entire land area of the continental United States?

7. The table given below gives the median age of males at the time of their first marriage from 1890-1998[6].

Year	Median age	Year	Median age
1890	26.1	1985	25.5
1900	25.9	1990	26.1
1910	25.1	1991	26.3
1920	24.6	1992	26.5
1930	24.3	1993	26.5
1940	24.3	1994	26.7
1950	22.8	1995	26.9
1960	22.8	1996	27.1
1970	23.2	1997	26.8
1980	24.7	1998	26.7

(a) Plot a graph showing median age of males at the time of their first marriage versus year.

(b) Based on the appearance of your plot, what kind of polynomial function (quadratic, cubic, etc.) would do a good job of representing the trends in the data? Explain how you used the appearance of the graph to decide on a polynomial function.

(c) Find an equation that could be used to predict the median age of males at the time of their first marriage given the year.

(d) Graph the polynomial function using the same set of axes as your plot of the data points. Where does the equation do a good job of representing the trends in the data?

(e) According to your function, what is the median age of males at the time of their first marriage in the year 2001?

(f) According to your function, what is the lowest median age during the twentieth century? What year did this minimum occur during? Does this minimum value match the data?

---

[5] Source: US Bureau of the Census. *Statistical History of the United States.*
[6] Source: US Bureau of the Census, *Marital Status and Living Arrangements* (1999) and *The Statistical History of the United States* (1976).

8. Let $f(x)$ be the function defined by the graph shown below and $g(x)$ the function defined by the table given below.

   Note: **1.** $f(x)$ is only defined for $-4 \le x \le 5$.

   2. $g(x)$ is only defined for $x = -4$, $x = -2$, $x = 0$, $x = 1$, $x = 2$ and $x = 3$.)
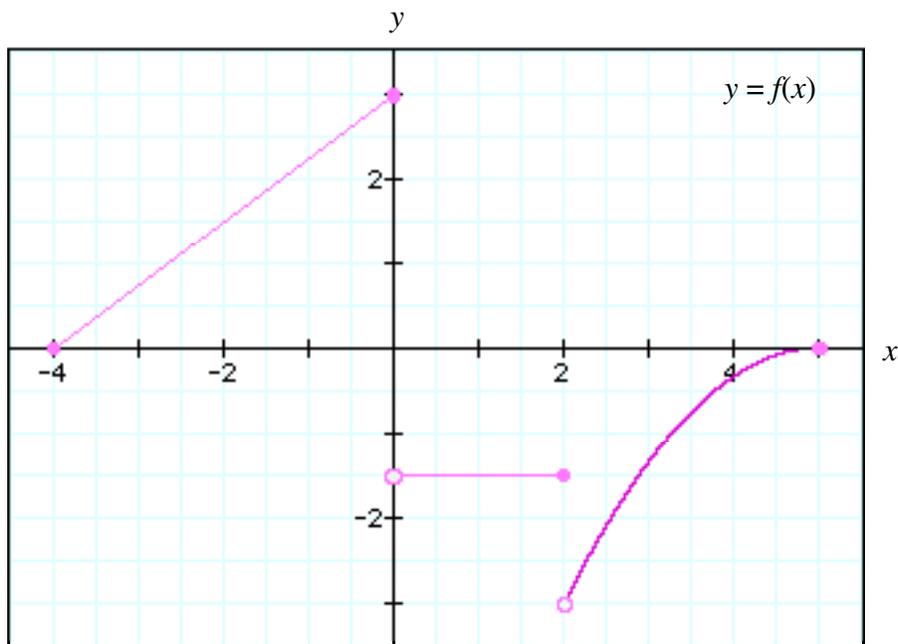
x	g(x)
−4	0
−2	−5
0	3
1	−1.5
2	0
3	−3

(a) Find all of the values of x for which: $f(x) = g(x)$.

(b) Find all of the values of k for which: $2 \cdot f(2) - f(1) = g(k)$.

(c) If possible, evaluate: $f^{-1}(-1.5) + g^{-1}(0)$. If you do not believe that it is possible to evaluate this expression, explain why.

9. During the 1970's and 1980's, the Chinese government surveyed a large amount of the Taklahmakhan Desert in Western China. In a portion of the desert known as the Tamin Basis, the surveyors found the mummified remains of people[7]. The mummies were Caucasian in appearance, and because the mummies were found near the ancient trade route known as the "Silk

---

[7] Source: http://www.discovery.com

Road" the remains were assumed to be the corpses of people who had died while travelling along this route.

(a)   Organic matter contains a radioactive isotope of carbon, carbon-14. A 100g sample of fresh organic matter will normally contain 0.0001 µg of carbon-14. Carbon-14 has a half life of 5730 years. Find an equation that will give the amount of carbon-14 that remains in a 100g sample of organic matter that is T years old.

(b)   Three of the mummies that were recovered were nicknamed "The Baby," "Cherchen Man" and "The Beauty of Loulan." During the 1990's, Japanese researchers paid over $100,000 to the Chinese government for 100g samples of these mummies. The amounts of carbon-14 found in these samples are shown in the table below. "The Baby" was a baby girl, "Cherchen Man" was an adult male standing over six feet tall, and "The Beauty of Loulan" was an adult woman. Based on the data given in the table below, could these three people have lived at the same time?

Mummy	Amount of carbon-14 found in 100g sample (µg)
The Baby	0.0000654
Cherchen Man	0.0000695
The Beauty of Loulan	0.0000616

(c)   Use the information contained in the table to work out how old each of the three mummies is.

(d)   The Silk Road was established after the Venetian traveler Marco Polo (1254-1324) made his historic journeys from Europe to the Far East. Is the theory that the mummies were unlucky travelers who died on the Silk Road correct or not? Briefly explain your reasoning.

10.   Let $h(x)$ be the function whose graph is shown below. The domain of the function $h(x)$ is the interval [0, 4].
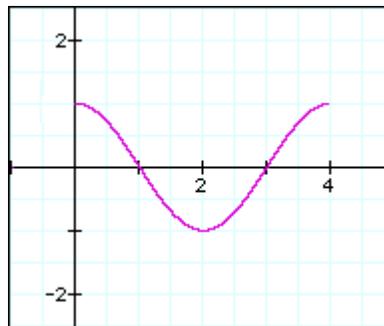
(a)   Over what intervals is $h(x)$ an increasing function? Over what intervals is $h(x)$ a decreasing function?

(b)   Over what intervals is $h(x)$ a concave up function? Over what intervals is $h(x)$ a concave down function?

(c)   Where are the zeros (i.e. the x-intercepts) of the function $h(x)$ located?

(d)   A new function $p(x)$ is defined by the equation:

$$p(x) = 2 \cdot h(-x) + 2.$$

What is the domain and range of the function $p(x)$?

(e)   Sketch an accurate graph showing $y = p(x)$. Where are the zeros of the function $p(x)$ located?