Practice Problems: Test #2, Set #3

Important Information:

1.   The first test will be held on **Tuesday December 3 from 7-9pm in Science Center D**.

2.   The test will include approximately eight problems (each with multiple parts).

3.   You will have 2 hours to complete the test.

4.   You may use your calculator and one page (8" by 11.5") of notes on the test.

5.   The specific topics that will be tested are:

- Functions defined in pieces.
- The idea of a limiting value.
- Left and right hand limits.
- Calculating limits of functions.
- Rational functions, asymptotes.
- Limits involving infinity.
- Average and instantaneous rate.
- Sketching the graph of a derivative.
- Calculating derivatives using limits.
- Graphical and verbal interpretations of the derivative.
- Short-cut rules for calculating derivatives.
- The product and quotient rules.
- Derivatives of exponential and logarithmic functions.
- Locating the maximum, minimum values of a function.

6.   The problems included here have been chosen because they are representative of many of the mathematical concepts that we have studied. There is no guarantee that the problems that appear on the test will resemble these problems in any way whatsoever.

7.   Good places to go for help include:

- Office hours.
- The labs on Tuesday 12/3.
- The Math Question Center
- The course-wide review on Monday evening. (Check course web site for exact time and location.)

8.   Remember: On exams, you will have to supply evidence for your conclusions, and explain why your answers are appropriate.
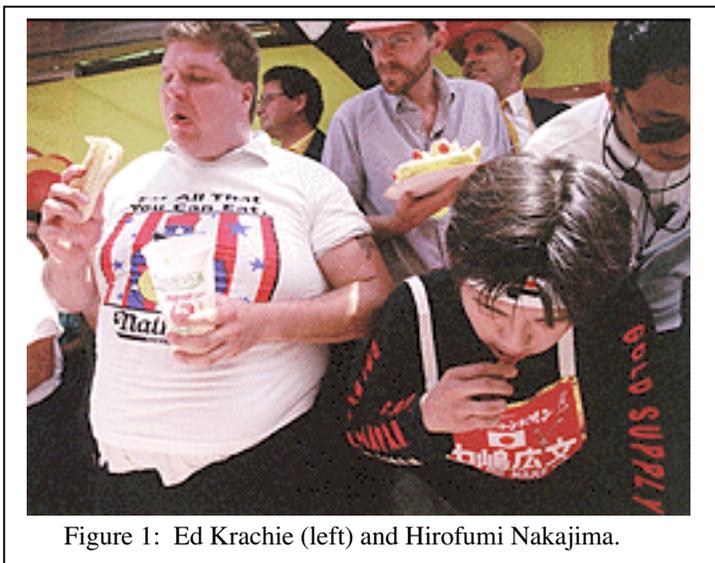
Figure 1: Ed Krachie (left) and Hirofumi Nakajima.

1. Every Fourth of July, the world championship of hot dog eating is held on Coney Island in New York City. Since July 6, 1996 (when 135 pound rank-outsider Hirofumi Nakajima beat the favored 381 Ed "The Animal" Krachie of Queens, New York - see Figure 1[1]) Japanese speed-eaters have had a dominated this event.

Last year, onlookers accustomed to Japanese supremacy in the field of hot-dog speed eating were again treated to an historic event when 131 pound Takeru Kobayashi (see Figure 2[2]) smashed the previous all-time world record[3] of 28.125 hot dogs by eating 50 hot dogs in 12 minutes[4].

The data for this accomplishment are given in the table below.

Figure 2: Takeru Kobayashi.

Time elapsed (minutes)	Number of hot dogs consumed by Takeru Kobayashi
0	0
3	16
4	21
8	33
11	44
12	50

(a) Draw a graph showing the number of hot dogs eaten by Mr. Kobayashi as a function of time. You can assume that the points of the graph are connected by straight-line segments.

(b) Let t denote the number of minutes that have elapsed in the competition, and $N(t)$ denote the number of hot dogs that Takeru Kobayashi has eaten. Find a collection of linear functions that you could use to construct a formula for $N(t)$.

(c) Before Takeru Kobayashi shattered the world record on July 4, 2001, the previous world record was 28.125 hot dogs in 12 minutes. According to your equations for $N(t)$, how many minutes did Takeru Kobayashi need in order to break the world record?

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[1] Image source: International Federation of Competitive Eating, http://www.ifoce.org/

[2] Image source: New York Daily News, July 5, 2001. http://www.nydailynews.com/

[3] Source: "Frankly, He's King of Coney." *New York Daily News*, July 5 2001. The previous record holder and former world hot dog eating champion was Kazutoyo "The Rabbit" Arai of Japan.

[4] This feat has been described by some semi-serious sport journalists as the greatest feat in the history of any athletic event. (See, for example, http://espn.go.com/gen/news/2001/0704/1222429.html) For comparison, Mr. Kobayashi's feat is comparable to: a professional baseball player hitting 140 home runs in a season OR a professional football player rushing for 550 yards in a single game OR a professional basketball player scoring 200 points in a single game.

(d)   The function you have defined should be an increasing function. Is it possible that this function could decrease, or is this (like some of the crime and tuna graphs that you have drawn) an example of a function that must increase no matter what?

2.   A tomato is thrown vertically into the air at time $t = 0$. Its height, $d(t)$ (in feet), above the ground at time t (in seconds) is given by the equation:

$$d(t) = -16 \cdot t^2 + 48 \cdot t.$$

(a)   Sketch a graph showing $y = d(t)$.

(b)   Find the times when the tomato's height above the ground is equal to zero. What is happening to the tomato the first time $d(t) = 0$? What is happening the second time?

(c)   When does the tomato reach its maximum height above the ground?

(d)   What is the maximum height of the tomato above the ground?

(e)   The equation for $d(t)$ is a quadratic function that has been written in standard form. Use the technique of completing the square to write an equation for $d(t)$ that is in vertex form.

(f)   How could you have used your answers to Parts (b), (c) and (d) to determine the vertex form of the equation for $d(t)$?
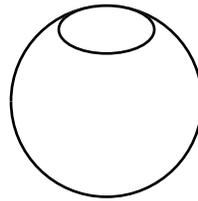
3.(a)   To help cover tuition and expenses, Marc works in a coffee shop. Through long and diligent practice, he has perfected the art of pouring coffee, so that the coffee runs into the cups at a constant rate. Match the graphs of the depth of coffee as a function of time with the four different shaped coffee cup. Be careful to explain why you think each choice is appropriate.
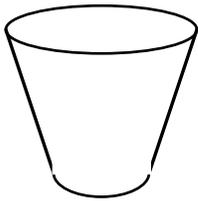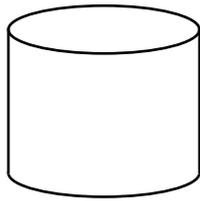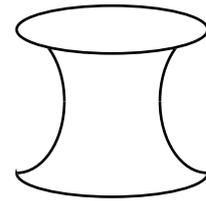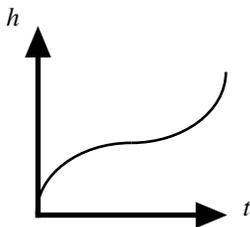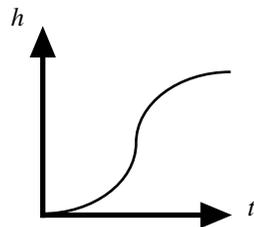
Cup 'A'   *Cup 'B'*   *Cup 'C'*   *Cup 'D'*
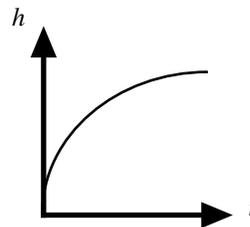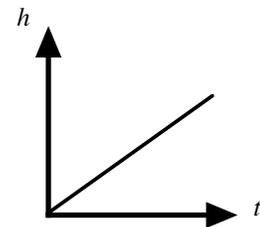
Graph 1   *Graph 2*   *Graph 3*   *Graph 4*

(b)   Let $d(t)$ be the depth of coffee (centimeters) in a coffee cup as a function of how much time Marc has been pouring for (in seconds). Write mathematical symbols that mean the same as:

It takes approximately 0.7 seconds for the depth of coffee in the cup to go from 10cm to 11cm.

4. In this problem, the functions f and g are defined by the equations given below.

$$f(x) = x^4 - x - 1 \qquad\qquad\qquad g(x) = \tfrac{1}{2}x^2 + 2$$

In this problem you will also be using a function h. All that you may assume about h is that:

- $h(2) = 1$
- $h'(2) = -1$.

(a) Define a new function k by the equation: $k(x) = f(x) \cdot g(x)$. Find an equation for the derivative $k'(x)$.

(b) Define a new function m by the equation: $m(x) = \dfrac{g(x)}{h(x)}$. Calculate the numerical value of the derivative $m'(2)$.

(c) Calculate an approximate value for $m(3)$.

5. In this problem, the functions f and g will always refer to the function defined by the equation:

$$f(x) = \tfrac{1}{2}x^3 - x^2 + 3\pi,$$

and the function whose derivative is graphed below.

$y = g'(x)$
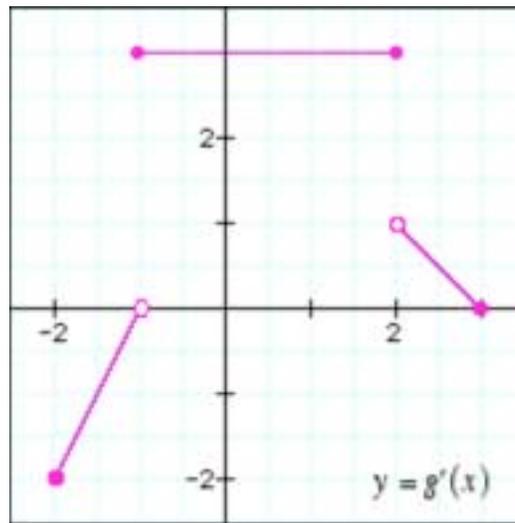
(a) Using any rules that you know for finding derivatives, find an equation for $f'(x)$.

(b) Let m be the function defined by the equation: $m(x) = f(x) + g(x)$. Find the value of $m'(0)$.

(c) Let r be the function defined by the equation: $r(x) = m'(x)$. Find an equation for $r(x)$ that is valid on the interval $0 \le x \le 1$.

6. In this problem, the function $f(x)$ always refers to:

$$f(x) = x^3 - 5x + 1.$$

(a) Find the critical point(s) of f.

(b) Where is f increasing and where is f decreasing?

(c) Where are the inflection points of f?

(d) Where is f concave up and where is f concave down?

(e) What is the global maximum (i.e. the very highest value) of f on the interval $[0, 5]$?

(f) What is the global minimum (i.e. the very lowest value) of f on the interval $[0, 5]$?

7. Let $P(t)$ represent the total petroleum reserves on Earth at the beginning of year 't.' $P(t)$ is measured in barrels. You may assume that no new petroleum is being made (except in the last question).

(a) What are the units of $\frac{dP}{dt}$?

(b) What is the sign of $\frac{dP}{dt}$ when $t = 1999$? Write a sentence or two explaining why your answer makes sense.

(c) Explain the meaning of the following algebraic expression in practical terms: $P'(1980) = R$.

(d) At the Motunui Synthetic Petrol plant in Taranaki, New Zealand, petroleum products are made out of natural gas (see Figures 3 and 4[5]). The rate at which petroleum is produced is too small to affect the sign of $\frac{dP}{dt}$. What effect does the Motunui plant have on the size of $\frac{dP}{dt}$?

Figure 3: The Synfuel facility located on the North Taranaki coast, near the town of New Plymouth.
Figure 4: An aerial photograph of part of the Synfuel facility.

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[5] Image source: Weake, H. J. (2000) "Taranaki - A good place to do business." in 2000 New Zealand Petroleum Conference Procedings, Crown Minerals, Ministry of Economic Development, P.O. Box M73, Wellington, New Zealand.

8. In this problem you are given four equations for rational functions and the graphs of five rational functions. Determine which equation matches which graph. (You should have one unmatched graph at the end of this problem.)

(I)

(II)

(III)

(IV)

(V)
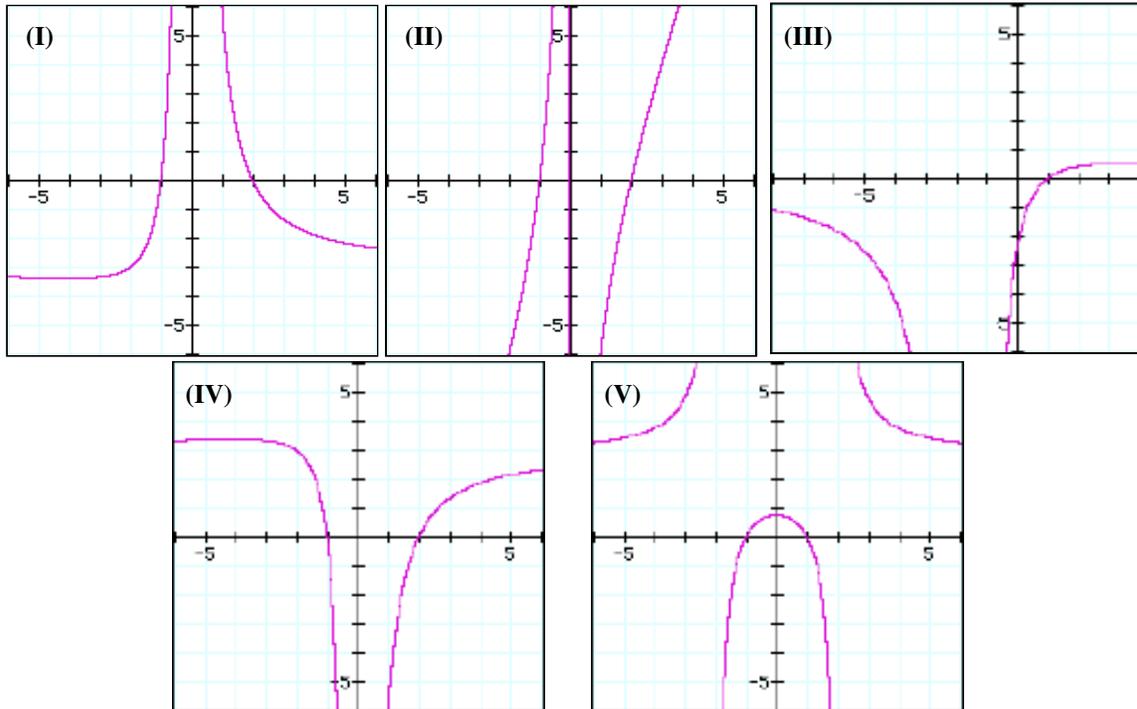
(a) $$y = \frac{3 \cdot (x + 1) \cdot (x - 2)}{x}.$$

(b) $$y = \frac{3 \cdot (x + 1) \cdot (x - 1)}{(x - 2) \cdot (x + 2)}.$$

(c) $$y = \frac{3 \cdot (x + 1) \cdot (x - 2)}{x^2}.$$

(d) $$y = \frac{5 \cdot (x - 1)}{(x + 1) \cdot (x + 2)}.$$

9. In this problem, the function $f(x)$ will always refer to the function defined by the equation:

$$f(x) = \frac{1}{x}.$$

(a) Set up the difference quotient of $f(x)$.

(b) Use the difference quotient that you found in Part (a) to find an equation for the derivative of $f(x)$.

(c) By re-writing the equation for $f(x)$, use one of the differentiation rules that you know to find an equation for the derivative of $f(x)$.

(d) Find an equation for the tangent line to the graph of $y = f(x)$ at the point (1, 1).

(e) Sketch a graph showing both $y = f(x)$ and the tangent line whose equation you found in Part (d).

10. In this problem, *f* is a function. All that you know about *f* is that:

$$f'(-1) = -2 \qquad f'(0) = 0 \qquad f'(1) = 2$$

Which of the following graphs could possibly be the graph of $y = f(x)$?

(a)   (b)   (c)   (d)