Name: _____

Class Time (circle one)   10   11   12

First Examination

Mathematics Xa

October 28, 1997

| Problem	Points	Score
1	11	
2	11	
3	14	
4	11	
5	11	
6	10	
7	6	
Total	100	

Please show all your work on this exam paper. You must show your work in order to get partial credit.
If you have work on the back of a page, indicate that on the exam cover.
Give exact answers except when an approximation is requested.
Work carefully, think clearly, and do well!

1. (11 points) The graph of f', *not* f is given below. The x-values a, b, c, d, e, g, and h are marked. Some of the questions below ask about f, others about f'. Please read carefully!
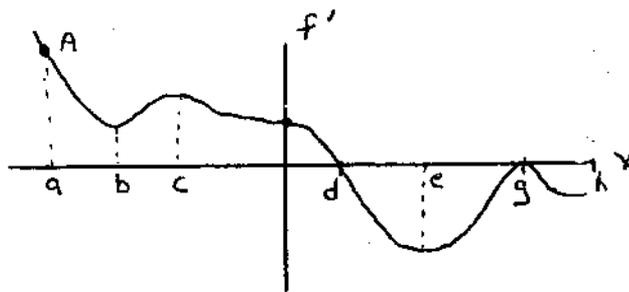

(a) Which is larger, $f(a)$ or $f(c)$? Explain briefly.

(b) Which is larger, $f(e)$ or $f(g)$? Explain briefly.

(c) Can you determine at which marked x-value $f(x)$ is greatest? Is so, tell us what it is. Explain your reasoning.

(d) Can you determine the largest value of f? If so, tell us what it is. Explain your reasoning.

(e) At which of the marked values of x is $f'(x)$ least? (This question is about f', the others ask about f.)

(f) On which interval(s) is f concave down and decreasing?

(g) At $x = a$ is f concave up or is f concave down?

2. (11 points) Below is a graph of $y = f(t)$. Some of the questions below ask about f, others ask about f'. Please read carefully.
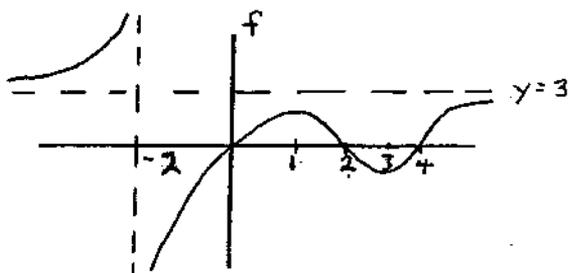
(a) On what interval(s) is f positive?

(b) On what interval(s) is f' negative?

(c) On what interval(s) is f decreasing?

(d) On what interval(s) is f' decreasing?

(e) On what interval(s) is f' both positive and decreasing?

(f) Sketch a graph of $f'(x)$.
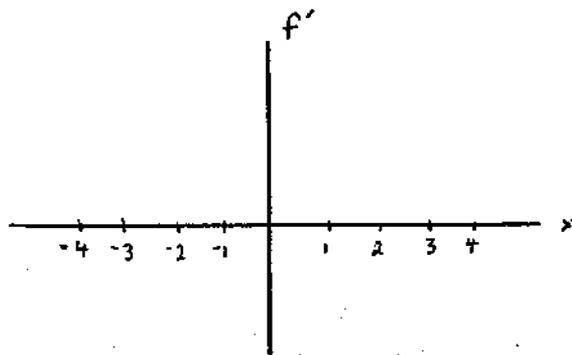
3. (14 points) Let

$$f(x) = \frac{x^2}{1 + x}.$$

(a) Use numerical methods to approximate $f'(3)$.

At $x = 3$ the graph of f is increasing and concave up. Give one approximation that is an upper bound for $f'(3)$, and another that is a lower bound for $f'(3)$.

Explain briefly how you *know* one is an upper bound and the other is a lower bound. (We suggest that you use a picture to explain this.)

(b) Using the limit definition of derivative, find $f'(3)$ exactly. You must produce an exact answer, not a numerical approximation. All your algebra must be explicitly demonstrated. (You need NOT find a general formula for $f'(x)$, only $f'(3)$.)

(c) Given your answer to (a), how much confidence do you have in your answer to (b)?

4. (11 points) A ferry is sailing back and forth on an East-West run across Puget Sound. Let $s(t)$ be the function that gives the ferry's position by indicating the number of miles from its western port as a function of time t, where t is measured in hours and $t = 0$ corresponds to noon.

For each of the phrases below write an equivalent mathematical expression in terms of the function s. *Include units in your answer.*

(a) the change in the ferry's position from 10 am to noon

(b) the ferry's average velocity between 2 pm and 4 pm

(c) the ferry's velocity at 4 pm

(d) the ferry's speed at 3 pm

For each of the sentences below write an equivalent mathematical equation in terms of the function s.

(a) At 5 pm the ferry is going 30 mph westward.

(b) At 9pm the ferry is 20 miles east of its western port.

5

5. (11 points) A technical typist works H hours per day D days per week. He spends p percent of his working time at home and the remainder in his office. He types at a rate of N pages per hour when he is at his office and types B pages every C hours when working at home. His wages are W dollars per week. Your answers should be in terms of H, D, p, N, B, C and W as appropriate.

   (a) How many hours per week does he spend working in his office?

   (b) How much does he earn per hour?

   (c) How much does he earn per page typed?

6. (10 points) At a certain time, which we will designate by $t = 0$, there were 1000 beavers in a certain region of Vermont. For the next 3 years the beaver population flourished, increasing by 10% every year. Then, with the advent of new road construction in the region, from time $t = 3$ on although the population continued to grow, it did so at a rate of 60 beavers every year.

(a) Write a function $B(t)$ giving the number of beavers at time t, t positive and measured in years. Please use a continuous model. (Note: your function will be defined differently for $0 \leq t \leq 3$ and for $t > 3$.)

(b) Use your function to determine the beaver population at time $t = 5$.

(c) Find $B'(5)$.

7. (6 points) An outdoor concert is being given. No rain is in the forecast, but a cold front is due to move in. The organizers expect attendence to be determined by the temperature at the start of the concert and have constructed a mathematical model. $A(F)$ is the number of people expected if the temperature is F where F is given in degrees Fahrenheit. Please interpret the following in words. Say precisely and unambiguously what you mean.

(a) $A(60) = 2030$

(b) $A'(55) = -40$