Name:

Section (circle one)

Robin's 10:00 Joe's 10:00 Andy's 11:00 Robin's 11:00 Eric's 12:00

First Examination

Mathematics Xa

October 26, 1998

Problem	Points	Score
1	16	
2	8	
4	10	
5	19	
6	12	
7	15	
8	11	
Total	100	

Please show all your work on this exam paper. You must show your work and clearly indicate your line of reasoning in order to get credit. If you have work on the back of a page, indicate that on the exam cover.

Give exact answers except when an approximation is requested.

You have two hours for this exam. Work carefully and efficiently. Think clearly and do well!

Mathematics Xa

First Examination

October 26, 1998

1. (16 points) A trolley travels back and forth along a straight track running north and south through a city. Let s be the function that associates with every time t the position of the trolley, where the benchmark position, $s = 0$, is the trolley stop at the city center. Positions north of the center are assigned positive values and those south are assigned negative values. Noon corresponds to the time $t = 0$. Position is measured in miles and time in hours.

Part I.

Under each expression (a) through (f), write the number corresponding to the equivalent mathematical expression from those displayed below.

(a) the trolley's position at 1:30 pm

_____

(b) the trolley's velocity at 1:30 pm

_____

(c) the change in the trolley's position between noon and 1:30 pm

_____

(d) the trolley's average velocity between noon and 1:30 pm

_____

(e) the change in the trolley's velocity from noon to 1:30 pm

_____

(f) the trolley's speed at 1:30 pm

_____

Mathematical Expressions

1. $\lim_{x \to 1.5} \dfrac{s(x) - s(1.5)}{x - 1.5}$
2. $\lim_{h \to 1.5} \dfrac{s(1.5 + h) - s(1.5)}{h}$
3. $s(1.5)$
4. $\dfrac{s(1.5) - s(0)}{2}$
5. $s'(1.5) - s'(0)$
6. $\left| \lim_{\Delta t \to 0} \dfrac{s(1.5 + \Delta t) - s(1.5)}{\Delta t} \right|$
7. $s'(1.5) - s'(0)$
8. $s(1.5) - s(0)$
9. $\dfrac{s(1.5) - s(0)}{1.5}$

Part II.

For each question below, write the equivalent mathematical statement. In other words, write the appropriate equation and indicate the variable that must be solved for.

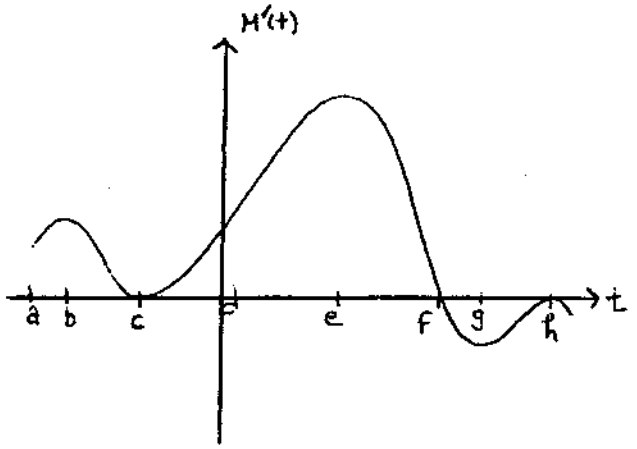
(a) At what time(s) is the trolley two miles south of the city center?

Equation: _____ Solve for what variable? _____

(b) When is the trolley going south at 2 miles per hour?

Equation: _____ Solve for what variable? _____

2. (8 points) Suppose we model the number of mosquitos within a certain region of a Louisiana bayou with the continuous function $M(t)$ where $M(t)$ gives the number of mosquitos as a function of time. Below is a graph of $M'(t)$, *NOT* $M(t)$. Use the graph to answer the questions that follow.



(a) At which point labeled is the population of mosquitos greatest? _____

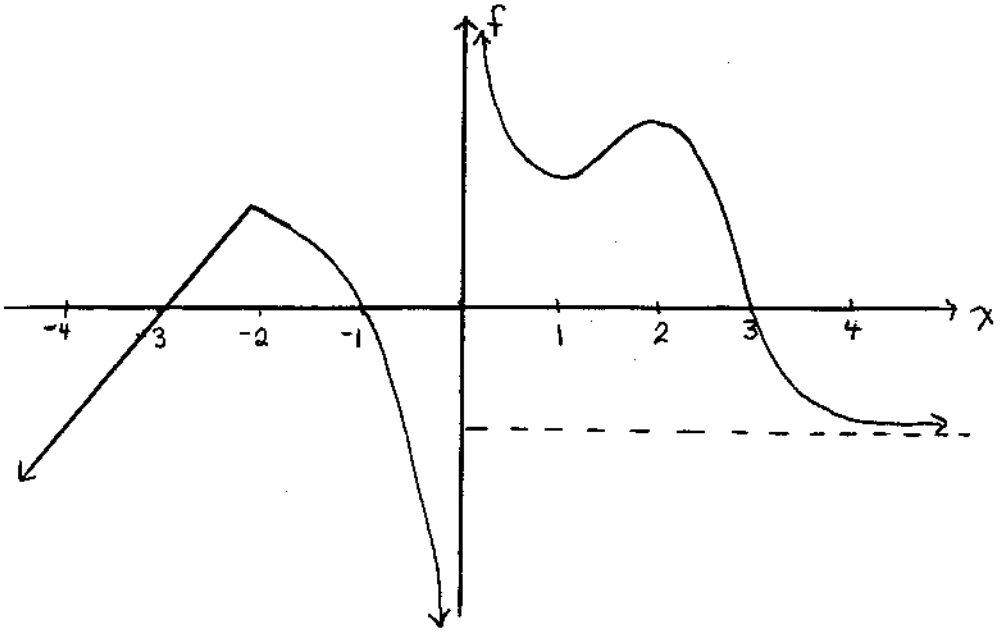
(b) At which point labeled is the population of mosquitos increasing most rapidly? _____

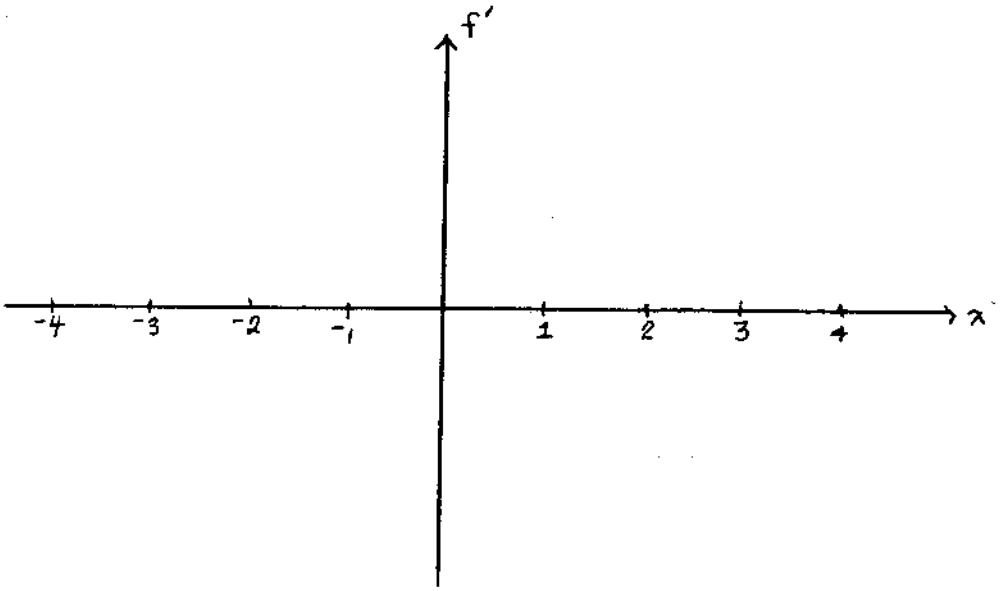
(c) At which point labeled is the population of mosquitos dropping most rapidly? _____

(d) At which point(s) labeled does the graph of $M(t)$ (*not* $M'(t)$) have a horizontal tangent line? _____

(e) Is $M'(t)$ a function? Explain why or why not.

4. (10 points) Given the graph of f below, sketch the graph of f' on the set of axes provided.





5. (19 points) Let f be the function

$$f(x) = \frac{x}{x-1}.$$

(a) What is the domain of f?

(b) Using the limit definition of derivative, find $f'(x)$.
We are not simply interested in your answer, but will also pay attention to your write-up.

(c) Based on your answer to part (b), is the function f increasing or decreasing on its domain? Explain briefly.

(d) Using your answer to (b), find $f'(3)$. How can you check whether or not your value for $f'(3)$ is in the right ballpark independent of your work in (b)?

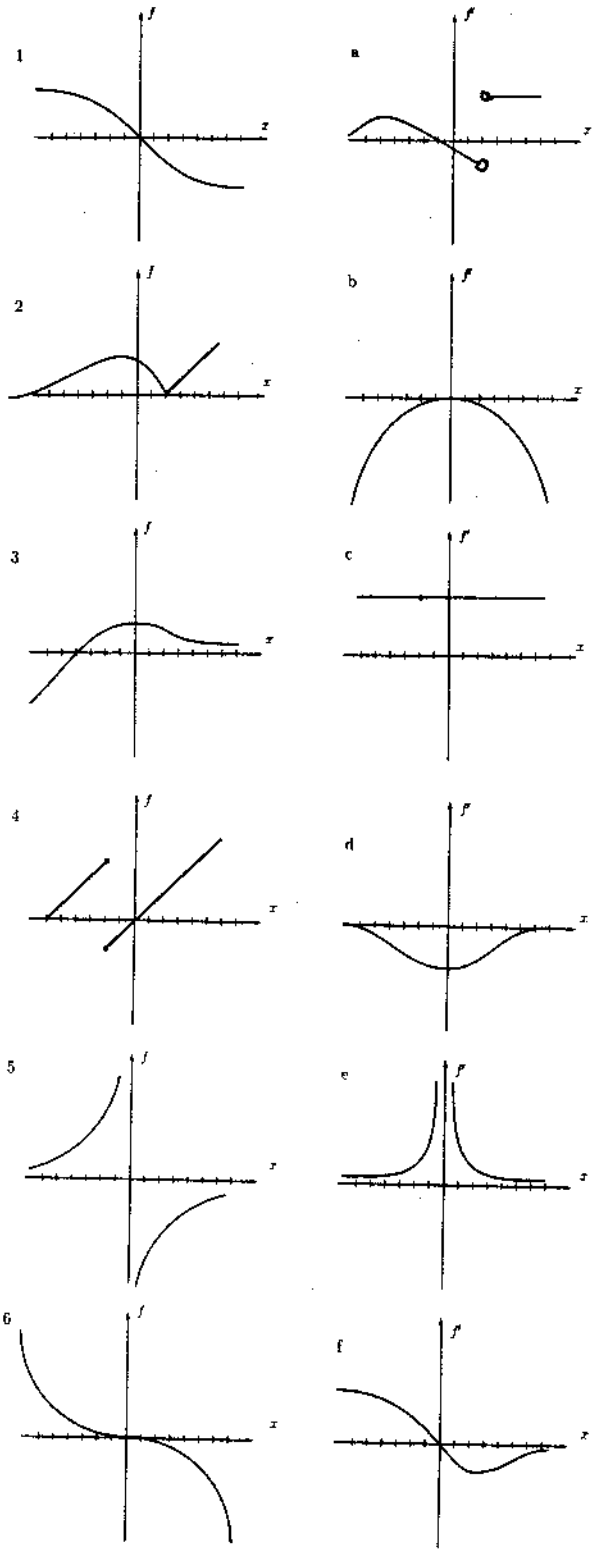
(e) What is the equation of the tangent line to $f(x)$ at $x = 3$?

6. (12 points) On the left below are the graphs of six functions, labelled 1 through 6, and on the right six functions labelled (a) through (f). For each of the first six, say which of the second six might be the graph of its derivative.

1. _____ 2. _____

3. _____ 4. _____

5. _____ 6. _____



7. (15 points) We want to buy a set of monogrammed pens so we check with several monogramming companies and get the following price quotes.

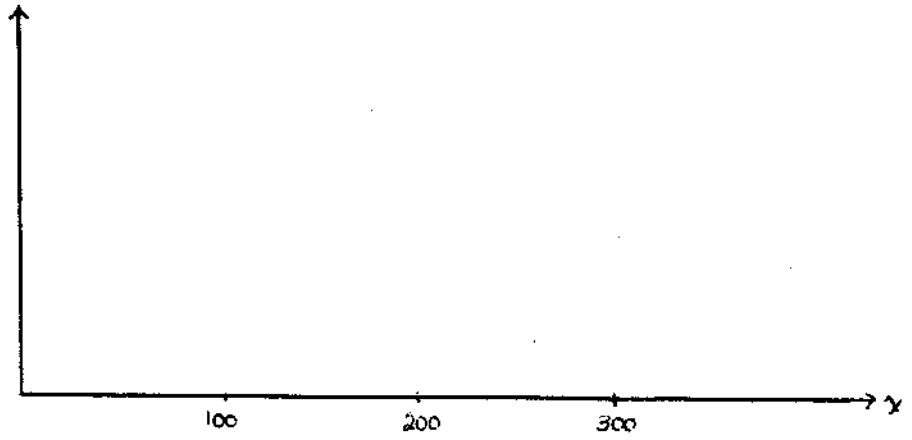
Pens-R-Us will charge \$20 just to place the order (they have to set up their specialized labelling machine for each order they process), and then they charge an additional 10 cents for each pen that we order.

Monogram Inc. will only charge \$10 to process the order, but then has a varying scale of costs per pen. For the first 100 pens we order, the cost is 30 cents per pen; for the next 100 pens the cost is 15 cents per pen, and for all pens after that the cost is 10 cents per pen.

Let $P(x)$ be the cost of ordering x pens from Pens-R-Us.

Let $M(x)$ be the cost of ordering x pens from Monogram, Inc.

(a) On the axes provided graph $P(x)$ and $M(x)$.



(b) Write functions for $P(x)$ and $M(x)$.

(c) For which values of x it is cheaper to order from Pens-R-Us as compared to ordering from Monogram Inc.?

(d) How much can the difference in prices between the two companies ever be if we place an order for the same number of pens from each company?

8. (11 points) At a county fair, a hot-air balloonist offers rides to the public. Let $C(A)$ be the amount (in dollars) she charges for a ride to an altitude of A feet. What is the meaning of each of the following? Your answer should be in plain English, in terms of cost and altitude. Explain carefully and precisely.

(a) The domain of the function is $[100, 600]$.

(b) $C(200) = 30$

(c) $C'(200) = .15$

(d) $C'(A) > 0$ for all $A > 100$.