Name:

Section (circle one)

Robin's 10:00 Joe's 10:00 Andy's 11:00 Robin's 11:00 Eric's 12:00

Second Examination

Mathematics Xa

December 8, 1998

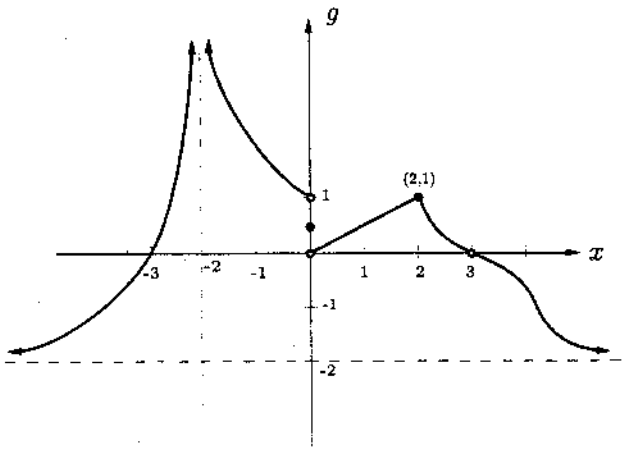
Problem	Points	Score
2	12	
3	20	
4	9	
5	16	
6	10	
7	14	
Total	100	

Please show all your work on this exam paper. You must show your work and clearly indicate your line of reasoning in order to get credit. If you have work on the back of a page, indicate that on the exam cover.

Give exact answers except when an approximation is requested.

You have two hours for this exam. Work carefully and efficiently. Think clearly and do well!

2. (12 points) Evaluate the following limits using the graph of g drawn below. Note: some of the limits ask about g, while others ask about g'. The domain of g is all real numbers except $x = -2$ and $x = 3$.



(a) $\lim_{x \to -2^+} g(x)$

(b) $\lim_{x \to 0^-} g(x)$

(c) $\lim_{x \to 0} g(x)$

(d) $\lim_{x \to \infty} g(x)$

(e) $\lim_{x \to 2} g(x)$

(f) $\lim_{x \to 3} g(x)$

(g) $\lim_{x \to \infty} g'(x)$

(h) $\lim_{x \to 2^-} g'(x)$

(i) $\lim_{x \to 2} g'(x)$

(j) $\lim_{x \to -2} g'(x)$

(k) At what points in the domain of g is $g'(x)$ undefined?

3. (20 points) Let $f(x) = 3x^4 - 4\pi x^3$ on the domain $(-\infty, \infty)$. All the questions that follow require exact answers, not numerical approximations.

(a) Identify all critical points of f.

(b) Identify all extrema of f, distinguishing between maxima and minima.
Put your answers in the blanks below. A given blank may have zero, one, or more answers; we want to you list all possible answers.

local minima :_____
local maxima :_____
Does f have an absolute minimum?_____
If so, what are the x- and y-coordinates of the absolute minimum? _____
Does f have an absolute maximum? _____
If so, what are the x- and y-coordinates of the absolute maximum? _____

(c) Identify the x-coordinate of all points of inflection. How can you be sure that these are points of inflection? How can you be sure that there are no others?

(d) At each point of inflection determine the slope of the tangent line to the graph of f at the point of inflection.

(e) Sketch a graph of $f(x)$ labelling all x-intercepts and the x-coordinates of all local extrema and all points of inflection.

(f) How many solutions are there to the equation $f(x - \pi) + 4 = 0$? Explain your reasoning, but *do not* try to find the solutions.

4. (9 points) $f(x)$ is the function given by $f(x) = ax^3 + bx^2 + cx + d$ where a, b, c, and d are constants. The constants b and d are positive and a and c are negative. The domain is $(-\infty, \infty)$.

   Circle the letter corresponding to every statement below that is undoubtedly <u>true</u>.

   (a) f is always negative.

   (b) f has an absolute maximum value but not an absolute minimum value.

   (c) f is continuous on $(-\infty, \infty)$.

   (d) $\lim_{x \to \infty} f'(x) = \lim_{x \to -\infty} f'(x)$

   (e) $\lim_{x \to -\infty} f(x) = -\infty$

   (f) f has at most two turning points.

   (g) f is positive and decreasing at $x = 0$.

   (h) f is differentiable everywhere on $(-\infty, \infty)$.

   (i) $\lim_{x \to -\infty} [f(x)]^2 = -\infty$

5. (16 points) Praja has invested \$3000 in a project; his investment is growing at a rate of 15% every two years. At the same time, his friend Dipendra invested \$2000; Dipendra's investment is growing by 9% per year.

   (a) After how many years will the value of Dipendra's investment surpass the value of Praja's investment? Give an exact answer and then give a numerical approximation.

   exact answer: _____ numerical approximation: _____

   (b) Exactly five years after his initial investment, how fast is Dipendra's investment growing? Give an exact answer and a numerical approximation. Include units in your answer.

   (c) At what time t is Dipendra's investment growing at a rate of 600 dollars /year? Give an exact answer.

6. (10 points)

(a) The function g with domain $(-\infty, \infty)$ is continuous everywhere. We are told that $g'(\sqrt{5}) = 0$. Some of the scenarios below would allow us to conclude that g has a local minimum at $x = \sqrt{5}$. Circle the letters corresponding to *all* such scenarios.

i. $g''(\sqrt{5}) > 0$
ii. $g''(\sqrt{5}) < 0$
iii. $g(\sqrt{5}) > 0$ and $g''(\sqrt{5}) = 0$
iv. $g(\sqrt{5}) < 0$ and $g'(x) > 0$ for all $x > \sqrt{5}$.
v. $g'(x) < 0$ for all $x < \sqrt{5}$ and $g'(x) > 0$ for all $x > \sqrt{5}$
vi. $g(\sqrt{5}) = 0, \quad g(2) = 1, \quad g(3) = 1$

(b) The function h with domain $[-8, -3]$ has the following characteristics.

- h is continuous at every point in its domain.
- $h'(x) > 0$ for $-8 < x < -4$ and $h'(x) < 0$ for $-4 < x < -3$.
- $h'(-4)$ is undefined.

What can you conclude about the local and absolute extrema of h? Please say as much as you can given the information above.

7. (14 points) A toothpick company decides to sell cardboard boxes containing 10 cubic inches of toothpicks. They would like to package them in boxes which use a minimum amount of cardboard. If one side of the toothpick box has to be exactly 2 inches long (the length of the toothpicks), then find the lengths of the other two sides of the box. (Use either the first or second derivative test to prove that you've found a minimum, not a maximum.)