Name:_____

Section (circle one)

Robin Gottlieb's 10:00     Andy Engelward's 11:00     Robin Gottlieb's 11:00     Tammy Lefcourt's 12:00

**Second Examination**

Mathematics Xa

December 6, 1999

| Problem | Points | Score |
|---------|--------|-------|
| 1 | 8 | |
| 2 | 8 | |
| 3 | 11 | |
| 4 | 7 | |
| | | |
| | | |
| 7 | 10 | |
| 8 | 7 | |
| | | |
| 10 | 12 | |
| 11 | 9 | |
| Total | 100 | |

Please show all your work on this exam paper. You must show your work and clearly indicate your line of reasoning in order to get credit. You can use a calculator, but "because my calculator says so" does not constitute reasoning. If you need to work on the back of a page, indicate that on the exam cover.

Give EXACT answers (as opposed to numerical approximations from your calculator) except when an approximation is requested.

You have two hours for this exam. Work carefully and efficiently. Think clearly and do well!

# Mathematics Xa

## Second Examination

### December 6, 1999

1. (8 points) Suppose $g(x)$ is a polynomial of even degree with a leading coefficient of -3. And suppose that $g'(e) = 0$ and $g''(e) = 4$.

   Circle **all** of the statements that are true. If more than one statement is true, then more than one statement should be circled. Briefly **explain** your rationale for circling the chosen statement.

   (a) $g$ has a point of inflection at $x = e$.

   (b) $x = e$ is a critical point of $g$.

   (c) $g$ has a local maximum at $x = e$.

   (d) $g$ has a local minimum at $x = e$.

   (e) $g$ might have an absolute maximum at $x = e$; we don't have enough information to say for sure.

   (f) $g$ might have an absolute minimum at $x = e$; we don't have enough information to say for sure.

2. (8 points) $Q(x)$ is a rational function whose graph has the following features:

   - vertical asymptotes at $x = -1$, $x = 0$, and $x = 1$
   - exactly one $x-$ intercept, at $x = 2$
   - $Q(x)$ is negative for all $x \in (-1, 1)$ and non-negative everywhere else
   - $\lim_{x \to \infty} Q(x) = 1$ and $\lim_{x \to -\infty} Q(x) = 1$.

   Find a possible equation for $Q(x)$.

3. (11 points) The graph of $f'$ (*not* $f$, but $f'$) is a parabola with $x$-intercepts of $-\pi$ and $2\pi$ and a $y$-intercept of $-2$.

(a) Draw a graph of $f'$.

$f'$

$x$
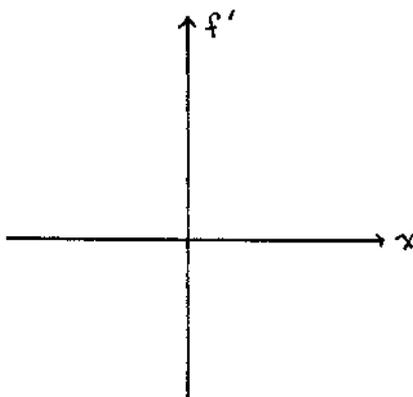
(b) Write an equation for $f'$. This equation should have no unknown constants.

(c) On the graph you drew in (a), go back and label the $x-$ and $y-$ coordinates of the vertex.

(d) Find $f''(x)$.

(e) This part of the question asks about $f$, *not* $f'$.

    i. Where does $f$ have a local maximum? Explain your reasoning clearly and briefly.

    ii. Where does $f$ have a local minimum? Explain your reasoning clearly and briefly.

    iii. Does $f$ have an absolute maximum or minimum value? Explain.

4. (7 points) Water is pouring continuously into a reservoir beginning at time $t = 0$. Let $W(t)$ be the number of gallons of water in the reservoir at time $t$, where $t$ is measured in seconds. Interpret each of the following in words. Your answers should be in plain English and should refer to gallons of water and seconds.

(a) $W(100)$

(b) $W'(200) = \frac{1}{2}$

(c) $W^{-1}(150)$

7. (10 points)

Consider the function $f(x) = x^5 - 2x^4 - 7$ restricted to the domain $[-1,1]$. Your reasoning for the questions below must be fully explained and be independent of a graphing calculator.

(a) Find the absolute maximum value of $f(x)$ on the interval $[-1,1]$ or explain why this is not possible.

(b) Find the absolute miniimum value of $f(x)$ on the interval $[-1,1]$ or explain why this is not possible.

(c) Find the absolute minimum value of $f(x)$ on the open interval $(-1,1)$ or explain why this is not possible.

8. (7 points)

Consider the graphs below. Circle the letters of the graph(s) for which $\lim_{x \to 3} f(x) = 2$.

A.



B.



C.



D.



E.



F.



G.
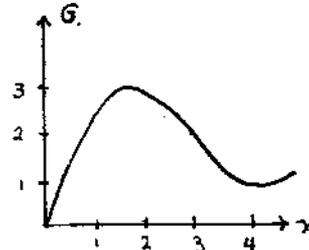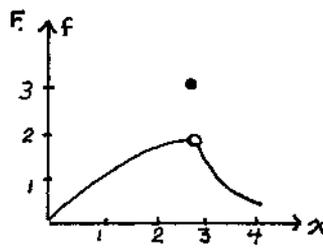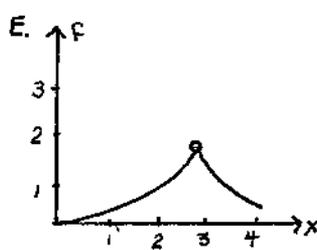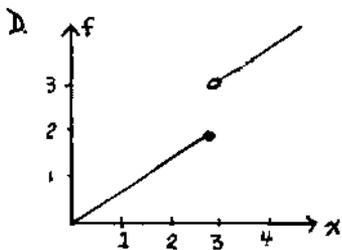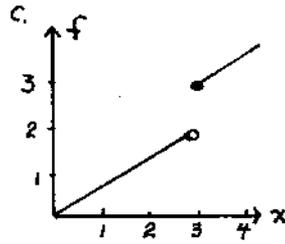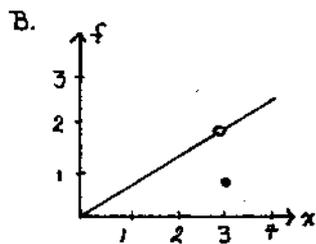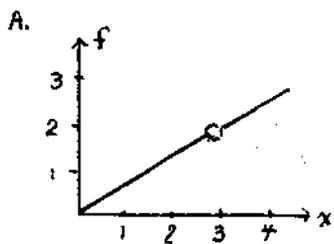
10. (12 points) Susan walks into her Chem lab one day and notices a vial of radioactive substance sitting on a lab scale. Taking into account the weight of the vial, she notes that the weight of the radioactive material is exactly 271 grams. Exactly one week later Susan walks back into the same lab, and sees the same vial of material sitting on the lab scale, having been undisturbed for the whole week. She notes that there are now just 138 grams of radioactive material. Susan has forgotten whether radioactive material decays exponentially or linearly.

(a) Give two equations giving the amount of radioactive material left t days after Susan's first visit to the Chem lab. One equation should model the decay as if it were a linear function (constant slope), and the other should model the decay as an exponential decay.

(b) Graph both of the functions from part (a) on the same axes, for values of t between 0 and 15.


amt. of radioactive material
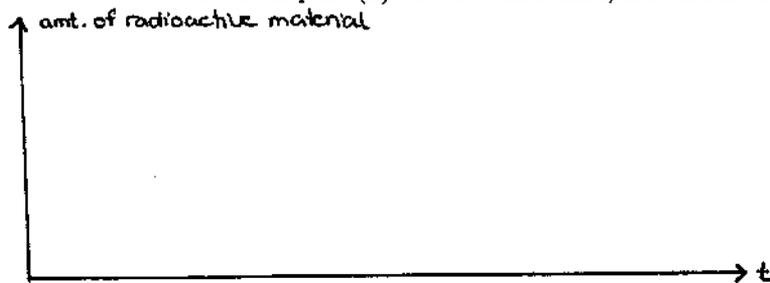
(c) According to the linear model, at what point will all of the substance have decayed (i.e. when will there be 0 grams of substance left)? According to the exponential decay model, at what point will all of the substance have decayed?

(d) What is the instantaneous rate of decay predicted by each model when t = 3 days? Bive both an exact answer and a numerical approximation.

11. (9 points) A photographer wants to frame an 8" × 10" photo with a mat. She wants to use a total of 200 square inches of matting material, with $x$ inches of matting above and below the photograph and $y$ inches of matting to the left and right of the photo (Note: The matting is indicated by the shaded region on the accompanying picture; there is no matting underneath the photo). The framing material is sold by the linear foot and is quite expensive. Therefore, she wants to minimize the perimeter of the frame.

(a) Express the perimeter of the frame as a function of $x$ and $y$.
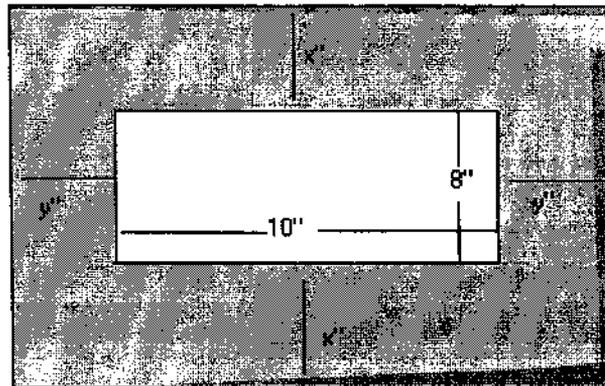


(b) Express the perimeter of the frame as a function of $y$ alone.

(c) Explain (but do not compute!) how to find the dimensions of the frame with minimum perimeter.

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