Some Review Problems for the Second Exam

(1) Sketch the graph of $y = 6/x^2 - 6/x$. While doing so answer the following questions.

a) What are the critical points of the function? Are there any local maxima or minima? Is there an absolute maximum value and minimum value for the function?

b) What are the points of inflection of the function?

Note: We strongly suggest that you draw three number lines, one indicating the sign of y, the next the sign of y ' and the third the sign of y ". Above the appropriate intervals on the number lines for the signs of y ' and y " indicate the significance of the sign for the graph of y.

(2) Graph $y = -5^{-x} - 3$.

(3) A tylenol capsule is formed by taking a cylinder of radius r and height h and capping both ends with hemispheres of radius r. What are the dimensions h and r which maximize the volume of the tylenol capsule if the surface area is fixed? Please explain how you know that the dimensions you've found actually *maximize* volume.

(4) An object is thrown straight up into the air at time $t = 0$. Its height is given by $3 + 40t - 16t^2$.

a) When is the object highest?

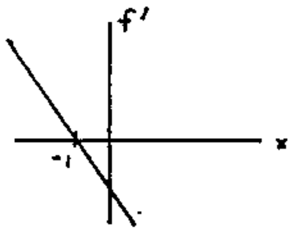
b) When does the object hit the ground ? (The ground is height zero.)

c) When is the speed of the object greatest?

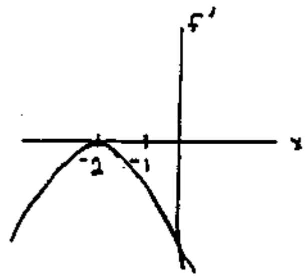
d) What is the object's acceleration? Does it change with time? (Does this make sense given what you know about physics?)

(5) Below are graphs of f '. For each, sketch the graph of f " and sketch a possible graph of f.

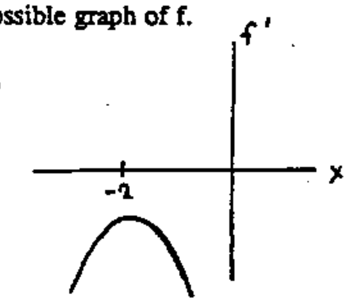
a)



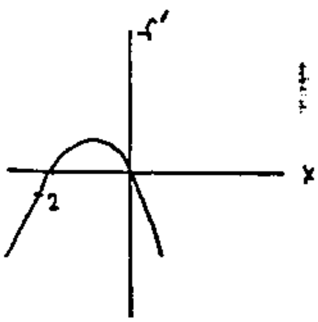
b)



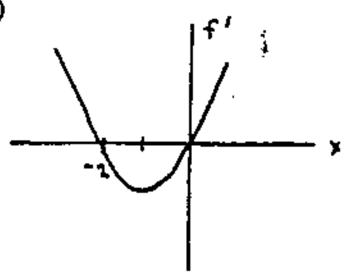
c)



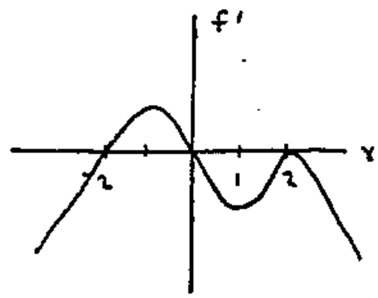
d)



d)



f)



(6) (12 points) a) Graph $y = x^3 - 6x^2 + 9x$. (without using your calculator) Clearly label all x-intercepts, extrema, and points of inflection. Indicate your reasoning clearly and fill in the blanks below:

local maximum point(s): ____________ local minimum point(s) : ____________

point(s) of inflection: ____________

b) Find the equation of the tangent line at the point of inflection. Draw this line on your graph.

(7) (12 points) Hialeah Racetrack in Miami, Florida, is famous for the flamingos that live in the infield. It is reasonable, if you are concerned about the living quaters of these flamingos, to be interested in making the rectangular part of the infield as large as possible. It is this concern that motivates our problem. For the purpose of horseracing, the perimeter of the racetrack is fixed.

The racetrack consists of a rectangular region capped by two semicircles. The perimeter, P, is fixed. Find the dimensions of the track (find x and y) that will maximize the area of the rectangular part of the infield. The rectangular part of the infield is shaded in the picture below.

Please tell us how you know that the dimensions you found actually ***maximize***the shaded infield area.

(For 1 point out of the 12): Suppose the perimeter is 7 furlongs, where a furlong is 1/8 of a mile. What is the area (in square miles) of the largest rectangular infield possible?

_________________ square miles

(8) (8 points) Suppose f is a continuous function whose domain is $(-\infty, \infty)$ and suppose that $f'(-2) = 0$ and $f''(-2) < 0$. Circle all of the statements that are true. (If more than one statement is true then more than one statement should be circled.)

a) -2 is a critical point of f but it is neither a local maximum point nor a local minimum point.

b) f has a local maximum point at x = -2

c) f has a local minimum point at x = -2

d) f has a point of inflection at x = -2

e) f definitely has a global maximum at x = -2

f) f definitely has a global minimum at x = -2.

g) possibly f has a global maximum at x = -2 , we don't have enough information to say for sure

h) possibly f has a global minimum at x = -2 , we don't have enough information to say for sure

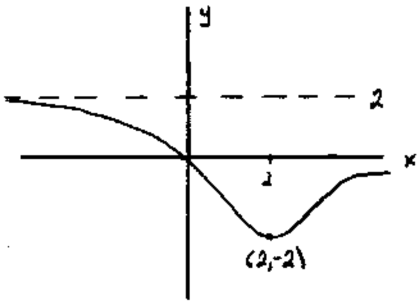
(9) 12 points) Two populations of bugs - green bugs and yellow bugs - are growing exponentially.

a) (1 point)At time t=0 there are 800 green bugs, and the population increases by 6% every week. Write the function G(t), the number of green bugs after t weeks.

(10) 12 points) Postal regulations require that the sum of the girth and the length of a parcel sent by parcel post may be no more than 108 inches. Find the dimensions of the cylindrical package of greatest volume that can be mailed by parcel post.
To earn full credit, you must give exact answers. Please show all your reasoning and present your work on the page in an organized way. Explain how you can be sure that the dimensions you found actually maximize the volume.

Note: The perimeter (circumference) of a circle is $2\pi r$. The area of a circle is πr^2.

(11) (6 points) Below is the graph of f(x).



Fill in the blanks. Choose from the graphs at the bottom of the page.

a) Graph # ____ is the graph of $y = f(x-2)$.

b) Graph #______ is the graph of $y = -f(x) + 2$.

c) Graph #______ is the graph of $y = f(2x)$.

d) Graph #______ is the graph of $y = f\left(\frac{x}{2}\right)$.