1. a) $\frac{d}{dx}[7xe^{x^3}] = 7\frac{d}{dx}[x\cdot e^{x^3}] = 7\cdot\left[\frac{d}{dx}(x)\cdot e^{x^3} + x\cdot\frac{d}{dx}(e^{x^3})\right]$ Fall 1998

$= 7\left[e^{x^3} + x\cdot e^{x^3}\cdot\frac{d}{dx}(x^3)\right] = 7\left[e^{x^3} + 3x^2\cdot x\cdot e^{x^3}\right]$

$= \boxed{7e^{x^3}(3x^3+1)}$

b) $\frac{d}{dx}\left[\frac{\sqrt{\ln(x^2)}}{5}\right] = \frac{\sqrt{2}}{5}\frac{d}{dx}\left[\sqrt{\ln(x)}\right] = \frac{\sqrt{2}}{5}\cdot\frac{1}{2}\frac{1}{\sqrt{\ln x}}\cdot\frac{d}{dx}[\ln(x)]$

$= \frac{\sqrt{2}}{10}\cdot\frac{1}{\sqrt{\ln x}}\cdot\frac{1}{x} = \boxed{\frac{\sqrt{2}}{10x\sqrt{\ln x}}}$

c) $\frac{d}{dx}\left[\frac{3^x}{\pi x^4}\right] = \frac{1}{\pi}\frac{d}{dx}\left[\frac{3^x}{x^4}\right] = \frac{1}{\pi}\left[\frac{\frac{d}{dx}[3^x]\cdot x^4 - 3^x\cdot\frac{d}{dx}(x^4)}{(x^4)^2}\right]$

$= \frac{\ln 3\cdot 3^x\cdot x^4 - 3^x\cdot 4x^3}{\pi x^8} = \boxed{\frac{3^x}{\pi x^5}((\ln 3)\cdot x - 4)}$

2. a) If the company spends half as much on advertising, they will make 80,000 less this year than last.

b) When \$30,000 is spent on advertising, an additional dollar spent would gross the company \$2.8

c) If the company made 2C dollars last year, they must have spent $R^{-1}(2C)$ on advertising.

3. $b$ = length of 1 side of the bottom, $s$ = height of sides.

$b^2\cdot s = 20\text{ ft}^3$. Cost $C = k[4bs] + 5k[b^2]$

$\frac{d}{db}\left[4kb\cdot\frac{20}{b^2} + 5kb^2\right] = -\frac{80k}{b^2} + 10kb = 0 \Rightarrow b^3 - 8 = 0 \Rightarrow \boxed{b=2}$

$C''(b) = \frac{160k}{b^3} + 10k$, which is positive for $b=2 \Rightarrow$ it's a minimum.

& since it's positive for $b > 0$, we know it's an absolute minimum.

So $\boxed{b=2,\ s=5}$

4. $f(x) = \frac{x^2}{e^x}$

a) $f'(x) = \frac{2x \cdot e^x - x^2 e^x}{e^{2x}} = \frac{2x - x^2}{e^x}$, which is positive for $(0 < x < 2) \Rightarrow \boxed{f \text{ is increasing for } 0 < x < 2}$

$f'(x)$ is negative for $x \in (-\infty, 0) \cup (2, \infty)$

$\Rightarrow f$ is decreasing for $\boxed{x \in (-\infty, 0) \cup (2, \infty)}$

b) $f''(x) = \frac{(2-2x)e^x - (2x - x^2)e^x}{e^{2x}} = \frac{x^2 - 4x + 2}{e^x} = 0$ @ $x = \frac{4 \pm \sqrt{16-}}{2}$

$= 2 \pm \sqrt{2}$

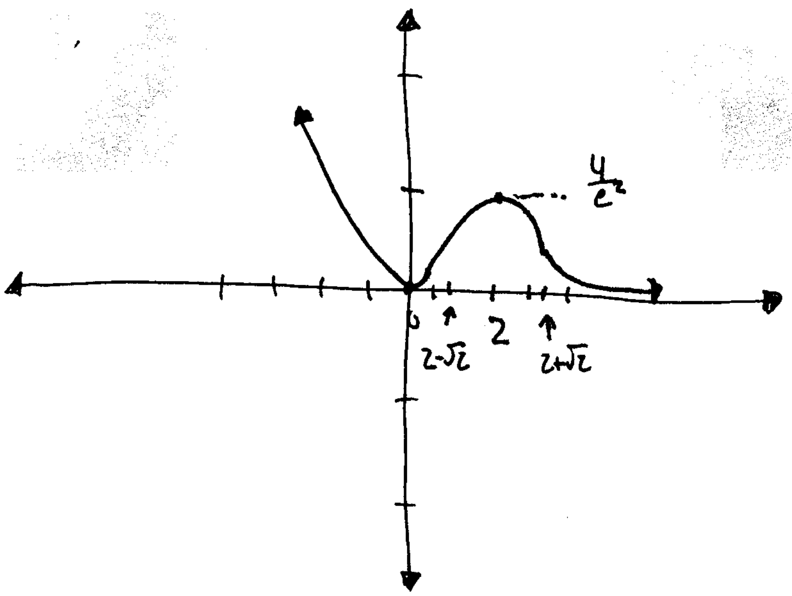
which is negative $\boxed{\text{(concave down) for } x \in (2-\sqrt{2}, 2+\sqrt{2})}$

and positive $\boxed{\text{(concave up) for } x \in (-\infty, 2-\sqrt{2}) \cup (2+\sqrt{2}, \infty)}$

c) $f(0) = 0$, $f(2) = \frac{4}{e^2}$, $\lim_{x \to -\infty} f(x) = \infty$, $\lim_{x \to \infty} f(x) = 0$

So $f$ has an absolute minimum at 0, & a relative max at 2.

d)




5a) $e^x + e^y = 3 \Rightarrow e^y = 3 - e^x \Rightarrow \boxed{\ln(3 - e^x) = y}$

5b) $2 + \ln(y^2) = x \Rightarrow 2\ln y = x - 2 \Rightarrow \boxed{e^{\frac{2x-4}{2}} = y}$

c) $\ln y + \ln 3y = x \Rightarrow \ln 3y^2 = x \Rightarrow 3y^2 = e^x \Rightarrow \boxed{y = \sqrt{3} \cdot e^{x/2}}$

d) $5y^3 \ln 2y = 0 \Rightarrow$ either $5y^3 = 0$ or $\ln 2y = 0 \Rightarrow \boxed{y = 0 \text{ or } \frac{1}{2}}$

6. $\boxed{\text{answers: b, f, h,}}$

7. a) $\lim_{x\to\infty} f(x) = \infty$ is sometimes true, depending on the sign of the coeff of the first term.

b) Always: $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) \Rightarrow$ it has to turn around somewhere i.e. it gets to a maximum height or max depth.

c) Sometimes: $x^2$ doesn't have an inflection point, $x^4$ does.

d) Sometimes: $f(x) = x^2$ works, $f(x) = x^2 - x$ doesn't.

e) Always.

f) Sometimes: local extrema occur when $f'(x) = 0$ & $f'$ can have at most $n-1$ real roots. It could have less, though.

g) Never: for $\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x)$, the graph must have an odd # of turnaround points. $n-2$ is even

8. i) a) just stretch it vertically by a factor of 2.

b) shift it 2 to the left.

c) stretch it horizontally by a factor of $\frac{1}{2}$

8(i) a) $a'(-2) = 2 \cdot f'(-2) = \boxed{3}$

b) $b'(-2) = f'(-2+2) = \boxed{0}$

c) $c'(-2) = 2 \cdot f'(2(-2)) = \boxed{1}$

^ this factor of 2 comes from the fact that when we half x, the $\frac{\text{rise}}{\text{run}}$ gets the run halved $\Rightarrow$ a factor of 2

That is – when we squish the graph, the slopes get steeper.

9. a) $f(x) = \frac{x}{x+1} \Rightarrow f'(x) = \frac{x+1-x}{(x+1)^2} = \boxed{\frac{1}{(x+1)^2}}$

b) $f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{\frac{2+h}{3+h} - \frac{2}{3}}{h}$

$= \lim_{h\to 0} \frac{6 - 6 - 2h}{h \cdot 3(3+h)} = \boxed{\frac{1}{9}}$

10. a) at $\boxed{t=3}$

b) at $\boxed{t=3}$

c) at $\boxed{t=-2}$

d) i) $s(t)$ is concave up when $s''(t) = v'(t)$ is positive

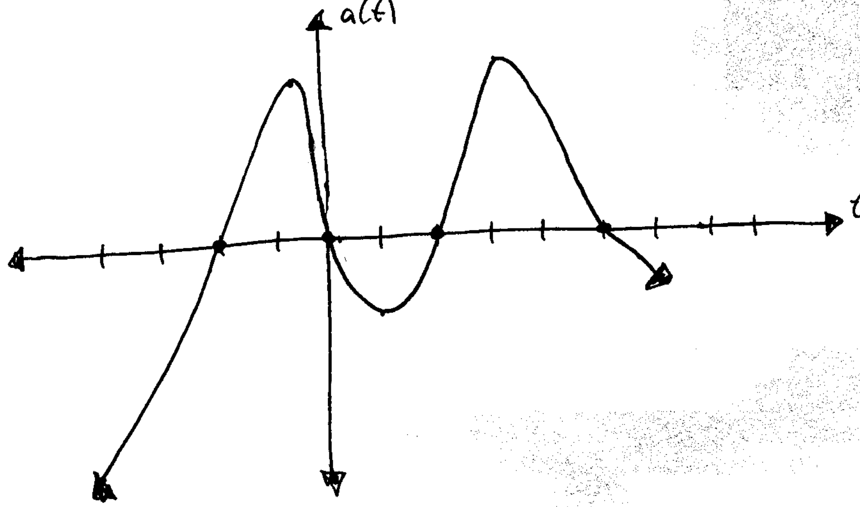
$\Rightarrow \boxed{t \in (-2,0) \cup (2,5)}$ as judged by the slope of $v(t)$

ii) $s(t)$ has an absolute minimum at $t=3$

$s(t)$ has no local minima, other than the absolute one.

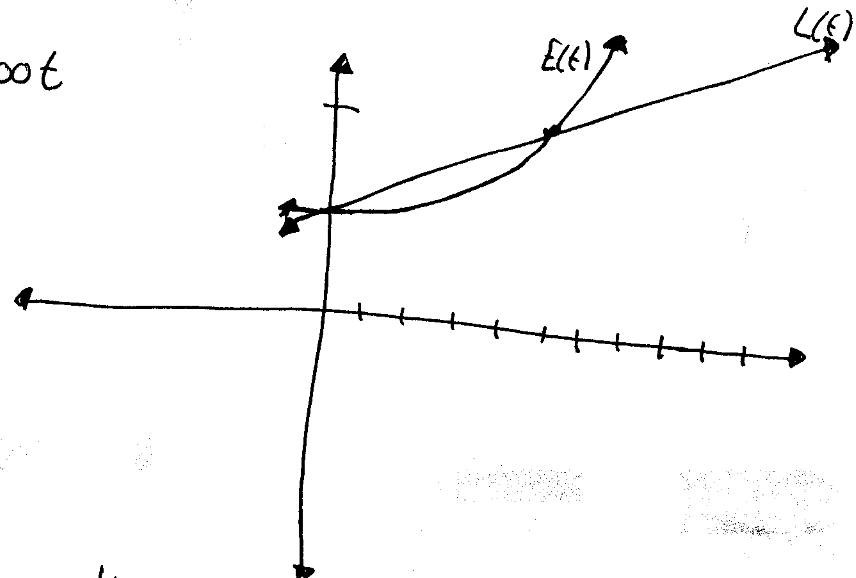
$s(t)$ has a point of inflection at $\boxed{t = -2, 0, 2, 5}$

10e)




11. a) $L(t) = 1000 + 200t$

$E(t) = 1000 \cdot 2^{\frac{t}{5}}$




b) Larry's grows more rapidly at $t=0$ (it grows at a rate of $200)

c) Earl's grows more rapidly at $t=5$ (it grows at a rate of

$E'(5) = 200 \cdot \ln 2 \cdot 2 = \boxed{400 \ln 2}$

d) iii They are growing at the same rate at $t_0$. If Larry's were growing faster, then a little later, the distance would be even greater, and if Earl's were growing faster, then a little earlier, the distance would have been greater.