Section 8.1 Local Linearity and the Derivative

Problem 1.

(a) $f(x) = \sqrt{x}$, $x = 25$.

Tangent line at $x = 25$ is $y - 5 = \frac{1}{2\sqrt{25}}(x - 25)$ or $y = 5 + 0.1(x - 25)$. So the linearization is $\sqrt{23} = f(23) \approx 5 + 0.1(23 - 25) \approx 4.8$. Using calculator we check that $\sqrt{23} \approx 4.795$

(b) $\sqrt{24} \approx f(24) = 0.1(24 - 25) + 5 = 4.9$; This is an overestimate, and using calculator we check that $|4.9 - \sqrt{24}| \approx |4.9 - 4.89898| = 0.00102$.

(c) $\sqrt{24.9} \approx f(24.9) = 0.1(24.9 - 25) + 5 = 4.99$; This is an overestimate, and using calculator we check that $|4.99 - \sqrt{24.9}| \approx |4.99 - 4.98999| = 0.00001$.

(d) $\sqrt{25.1} = f(25.1) \approx 5 + 0.1(25.1 - 25) \approx 5.01$. Using calculator we check that $\sqrt{25.1} \approx 5.01$

(e) $\sqrt{26} \approx f(26) = 0.1(26 - 25) + 5 = 5.1$; This is an overestimate, and using calculator we check that $|5.1 - \sqrt{26}| \approx |5.1 - 5.09902| = 0.00098$. $|5.1 - \sqrt{26}| \approx |5.1 - 5.09902| = 0.00098$.

(f) $\sqrt{27} = f(27) \approx 5 + 0.1(27 - 25) \approx 5.2$. Using calculator we check that $\sqrt{27} \approx 5.196$

Problem 5.

$f(x) = \sqrt[3]{x}$, $x = 27$.

The slope of the tangent to $f(x)$ is $(\frac{1}{3}x^{-\frac{2}{3}})|_{x=27} = \frac{1}{27}$. So, the tangent is $y - 3 = \frac{1}{27}(x - 27)$ or $y = 3 + \frac{1}{27}(x - 27)$.
Hence $\sqrt[3]{30} \approx 3 + \frac{1}{27}(30 - 27) \approx 3.111$
Using calculator we check that $\sqrt[3]{30} \approx 3.107$

Section 8.3 Derivatives of Sums, Products, Quotients, and Power Functions

Problem 1.

$f'(x) = 6x + 3 - 3x^{-2} - 6x^{-3}$

Problem 2.

$f(x) = \frac{1}{5}(x - 2x^2) \Rightarrow f'(x) = \frac{1}{5}(1 - 4x)$

Problem 3.

Product Rule. $f'(x) = \pi[(6x + 7)(x - 2) + (3x^2 + 7x + 1)] = \pi[6x^2 - 12x + 7x - 14 + 3x^2 + 7x + 1] = \pi[9x^2 + 2x - 13]$

Problem 4.
Quotient Rule. $f'(x) = \frac{0 \cdot (x^2+4) - 1(2x)}{(x^2+4)^2} = \frac{-2x}{(x^2+4)^2}$

Problem 5.
Quotient Rule. $f'(x) = \frac{(x+2)(1)-(x)(1)}{(x+2)^2} = \frac{2}{(x+2)^2}$

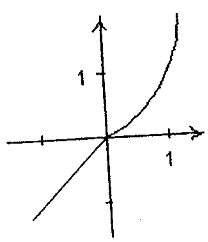
Problem 6.
$f(x) = \frac{x+2}{x} = 1 + 2x^{-1}, \quad f'(x) = -2x^{-2} = \frac{-2}{x^2}$

Problem 7.
$f(x) = \left(\frac{5}{2}x^2 + 7x^5 - 5x\right)x = \frac{5}{2}x^3 + 7x^6 - 5x^2 \Rightarrow f'(x) = \frac{15}{2}x^2 + 42x^5 - 10x.$

Problem 8.
$f(x) = ax^{-1} + bcx - bdx^2, \quad f'(x) = -ax^{-2} + bc - 2bdx = -\frac{a}{x^2} + b(c - 2dx)$

Problem 14.

(a)



(b) $f'(x) = \begin{cases} 2x & ,x > 0 \\ 1 & ,x < 0 \\ undefined & ,x = 0 \end{cases}$

Problem 17.
$f(x) = \begin{cases} x^3 & ,x \leq 1 \\ kx & ,x > 1 \end{cases}$

(a) To make f continuous at $x = 1$ we have to choose k such that $x^3 = kx$ when $x = 1$. So $k = 1$.

(b) f is not differentiable at $x = 1$.

Problem 20.
$\frac{d}{dx}\left(\frac{x+1}{x^3+3x+1}\right) = \frac{x^3+3x+1-(x+1)(3x^2+3)}{(x^3+3x+1)^2} = \frac{x^3+3x+1-3x^3-3x-3x^2-3}{(x^3+3x+1)^2} = \frac{-2x^3-3x^2-2}{(x^3+3x+1)^2}$

Problem 21.
$$\frac{d}{dx}\left(\frac{\pi}{\pi x + \pi}\right) = \frac{d}{dx}\left(\frac{1}{x+1}\right) = \frac{(x+1)(0) - (1)(1)}{(x+1)^2} = -\frac{1}{(x+1)^2}.$$

Problem 22.
$$\frac{d}{dx}\left(\frac{2x^2+x+1}{\sqrt{2x}}\right) = \frac{(\sqrt{2x})(4x+1) - \left(\frac{1}{\sqrt{2x}}\right)(2x^2+x+1)}{2x} = \frac{(2x)(4x+1) - (1)(2x^2+x+1)}{2\sqrt{2}x^{3/2}} =$$
$$\frac{8x^2+2x-2x^2-x-1}{2\sqrt{2}x^{3/2}} = \frac{\sqrt{2}(6x^2+x-1)}{4x^{3/2}}.$$

Problem 23.
$\frac{d}{dx}\left(\frac{x^2+5x}{2x^{10}}\right) = \frac{d}{dx}\left(\frac{1}{2}x^{-8} + \frac{5}{2}x^{-9}\right) = -4x^{-9} - \frac{45}{2}x^{-10} = -\frac{4}{x^9} - \frac{45}{2x^{10}}$