Section 9.1 Exponential Growth

Problem 1.

(a)

t	B(t)
1	600
2	1800
3	5400
4	16200
5	48600

(b) At noon yesterday it was 67, at noon four days ago 2.5 (approximately)

(c) $B(t) = B_0 \cdot 3^t$

(d) $B(w) = B_0 \cdot 3^{7w}$

(e) $B(1) = 200 \cdot 3^7 = 437400$

Problem 3.

(a) $E(h) = 600 \cdot 2^{3h}$, 20 minutes $= \frac{1}{3}$ hour

(b) $X(h) = 100 \cdot 2^{4h}$, 15 minutes $= \frac{1}{4}$ hour

(c) $E(h) = X(h)$ when $600 \cdot 2^{3h} = 100 \cdot 2^{4h}$, which is $6 \cdot 8^h = 16^h$ or $6 = 2^h$. So $h \approx 2.584$.
Answer: approximately after 2 hours and 35 minutes.

9.2

Section 9.2

Problem 8.

$$\frac{(ab)^{-x}}{a^{-x}+b^{-x}} = \frac{a^{-x}b^{-x}}{a^{-x}+b^{-x}} = \frac{1}{(ab)^x(a^{-x}+b^{-x})} = \frac{1}{a^xb^xa^{-x}+a^xb^xb^{-x}} = \frac{1}{b^x+a^x}$$

Problem 9.

$$\frac{\left(a^{1-x}b\right)^3}{\left(a^2b^3\right)^x} = \frac{a^{3-3x}b^3}{a^{2x}b^{3x}} = a^{3-5x}b^{3-3x}$$

Problem 25.

(a) $C > 0, \ a \in (0,1)$

(b) $C < 0, \ a \in (0,1)$

(c) $C < 0, \ a > 1$

(d) $C > 0, \ a > 1$

Problem 26.
Because $\lim\limits_{x\to-\infty} f(x) = 0$, $D = 0$. Now $3 = f(0) = C + D = C + 0 \Rightarrow C = 3$. Substituting the point $(1,5)$ into the formula for $f(x)$ gives $f(1) = 3a = 5$, from which we obtain $a = \frac{5}{3}$. Thus, $f(x) = 3\left(\frac{5}{3}\right)^x$.

Problem 27.
Because $\lim\limits_{x\to-\infty} f(x) = 2$, $D = 2$. Now $4 = f(0) = C + D = C + 2 \Rightarrow C = 2$. Substituting the point $(-1,3)$ into the formula for $f(x)$ gives $f(-1) = 2 \cdot \frac{1}{a} + 2 = 3$, from which we obtain $a = 2$. Thus, $f(x) = 2\left(2^x\right) + 2$.

Problem 28.
Because $\lim\limits_{x\to-\infty} f(x) = -1$, $D = -1$. Now $-3 = f(0) = C + D = C - 1 \Rightarrow C = -2$. Substituting the point $(1,-4)$ into the formula for $f(x)$ gives $f(1) = -2 \cdot a - 1 = -4$, from which we obtain $a = \frac{3}{2}$. Thus, $f(x) = -2\left(\frac{3}{2}\right)^x - 1$.

Problem 29.

Because $\lim_{x \to -\infty} f(x) = 0$, $D = 0$. Now $\frac{3}{2} = f(0) = C + D = C + 0 \Rightarrow C = \frac{3}{2}$. Substituting in the point $(2, \frac{1}{2})$ into the formula for $f(x)$ gives $f(2) = \frac{3}{2} \cdot a^2 = \frac{1}{2}$, from which we obtain $a = \frac{1}{\sqrt{3}}$. Thus, $f(x) = \frac{3}{2}\left(\frac{1}{\sqrt{3}}\right)^x$.

Problem 31.

(a) (i); (b) (ii); (c) (vi); (d) (iii); (e) (iii); (f) (v)