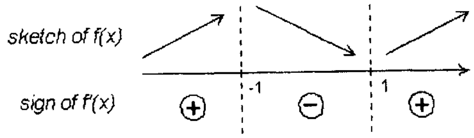
Section 10.1 Analysis of Extrema

Problem 1.

(a) $f'(x) = 3x^2 - 3$. $f'(x) = 0 \Leftrightarrow 3x^2 - 3 = 0$, so $x^2 - 1 = 0$ and therefore $x = 1$ or $x = -1$ are critical points.

(b)



This tells us that at $x = -1$ f has a local maximum and at $x = 1$ f has a local minimum.

(c) There are no absolute minimum or maximum values.

Problem 2.

(a) From problem 1, we have that $x = -1$ and $x = 1$ are critical points. The endpoints of the domain $[-5, 5]$, $x = -5$ and $x = 5$, are also critical points.

(b) From problem 1, $x = -1$ is a local maximum point and $x = 1$ is a local minimum point. The absolute minimum occurs at $x = -5$, and the absolute maximum occurs at $x = 5$.

(c) The absolute minimum value is $f(-5) = -108$, and the absolute maximum value is $f(5) = 112$.

Problem 3.

(a) From problem 1, we have that $x = 1$ is a critical point. The endpoints of the domain, $x = 0$ and $x = 3$, are also critical points.

(b) The point $x = 0$ is neither a local maximum or minimum point nor an absolute minimum or maximum point. From problem 1, $x = 1$ is a local minimum point and the absolute minimum point. The point $x = 3$ is the absolute maximum point.

(c) The absolute minimum value is $f(1) = 0$, and the absolute maximum value is $f(3) = 20$.

Problem 7.

(a) $f'(x) = 5x^4 - 20 = 5(x^4 - 4) \Rightarrow 0 = f'(x) = 5(x^2 - 2)(x^2 + 2) \Rightarrow x = -\sqrt{2}$ and $x = \sqrt{2}$ are the critical points.

(b) $f'(x)$ is positive on $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$ and is negative on $(-\sqrt{2}, \sqrt{2})$. The first derivative test implies that $x = -\sqrt{2}$ is a local maximum point and $x = \sqrt{2}$ is a local minimum point. As $f(x)$ increases without bound as as x increases without bound, $x = -\sqrt{2}$ is not an absolute maximum point. As $f(x)$ decreases without bound as as x decreases without bound, $x = \sqrt{2}$ is not an absolute minimum point.

(c) There are no absolute minimum or maximum values.

Problem 8.

(a) From problem 7, $x = -\sqrt{2}$ is a critical point. The endpoints of the domain $[-2, 0]$, $x = -2$ and $x = 0$, are also critical points.

(b) The point $x = -2$ is neither a local maximum or minimum point nor an absolute minimum or maximum point. The point $x = -\sqrt{2}$ is a local and and absolute maximum point. The point $x = 0$ is the absolute minimum point.

(c) The absolute minimum value is $f(0) = 5$, and the absolute maximum value is $f(-\sqrt{2}) = 16\sqrt{2} + 5$.

Problem 9.

(a) From problem 7, $x = \sqrt{2}$ is a critical point. The endpoints of the domain, $x = 0$ and $x = 2$, are also critical points.

(b) The point $x = 0$ is the absolute maximum point. The point $x = \sqrt{2}$ is a local and absolute minimum point. The point $x = 2$ is neither a local maximum or minimum point nor an absolute minimum or maximum point.

(c) The absolute minimum value is $f(\sqrt{2}) = 5 - 16\sqrt{2}$, and the absolute maximum value is $f(0) = 5$.

Problem 18.

(a) $f'(x) = 2xe^{-x} + x^2(-e^{-x}) = -x(x-2)e^{-x}$; $0 = f'(x) \Rightarrow x = 2$ and $x = 0$ are the critical points.

(b) $f'(x)$ is positive on $(0, 2)$ and is negative on $(-\infty, 0) \cup (2, \infty)$. The first derivative test implies that $x = 0$ is a local minimum point and $x = 2$ is a local maximum point.

(c) No. The function values of f increase without bound as x decreases without bound.

(d) Yes, the absolute minimum value of $f(0) = 0$ is achieved at $x = 0$.