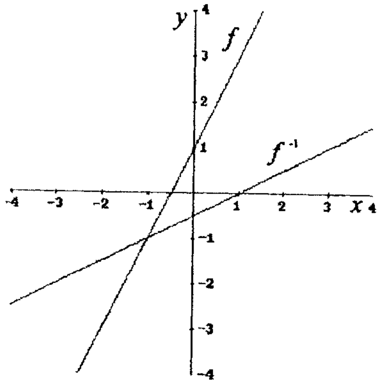
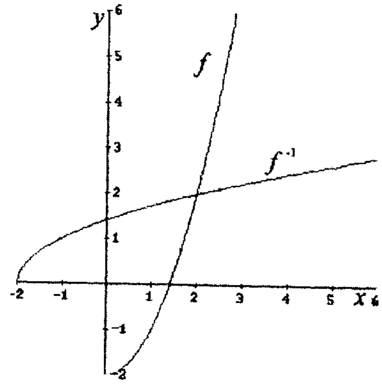
12.1

Problem 3.

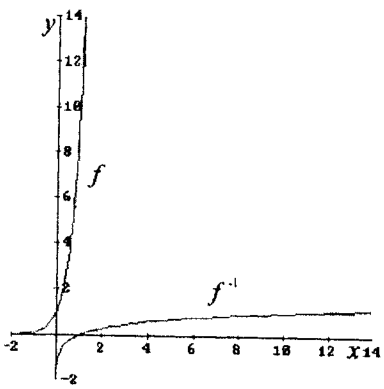
(a) $f^{-1}(x) = \frac{x-1}{2}$



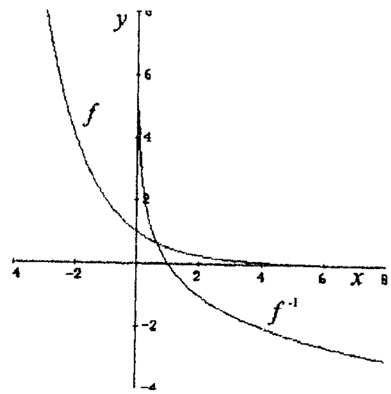
(b) $f^{-1}(x) = \sqrt{x+2},\ x > -2$



(c)



(d)



Problem 5.

(a) Not invertible, because $P(w)$ is a piecewise constant function. For example, all packages up to one ounce will have the same mailing price

(b) Not invertible, since the same temperature may occur on different days.

(c) Invertible: If $C(w) = kw$, for some positive constant k, then if $A(c)$ is the amount of coffee that can be purchased for c dollars, we have $A(c) = \frac{c}{k}$. The function $A(c)$ is the inverse of the function $C(w)$.

(d) Not invertible, because the car could sit in the garage for a few days.

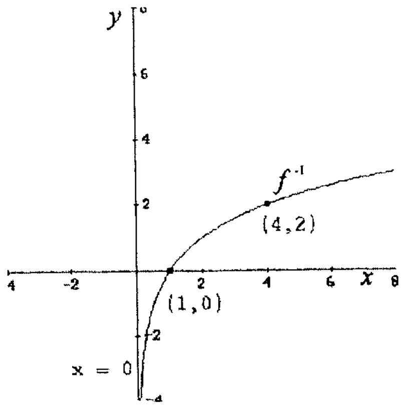
Problem 6.

(a) Consider $f'(x) = 3x^2 + 6x + 6$, which is defined and continuous for all real values of x. The discrimanant of $3x^2 + 6x + 6 = 0$ is $6^2 - 4(3)(6) = -36 < 0$; hence $f'(x)$ is never equal to zero. Moreover, as $f'(0) = 6 > 0$, f' is increasing everywhere; that is, if $a < b$, then $f(a) < f(b)$. Hence f is 1-to-1 and, therefore, invertible.

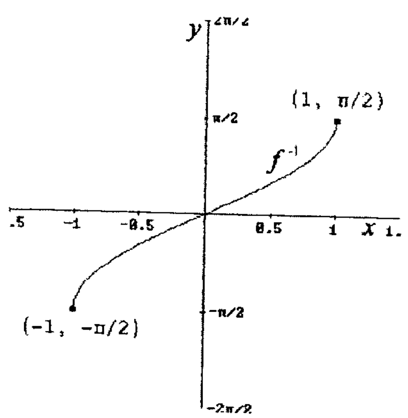
(b) Answers will vary. $(12, 0)$, $(22, 1)$, $(44, 2)$ are on the graph of $f^{-1}(x)$, because $(0, 12)$, $(1, 22)$, and $(2, 44)$ are points on the graph of $f(x)$.

Problem 7.

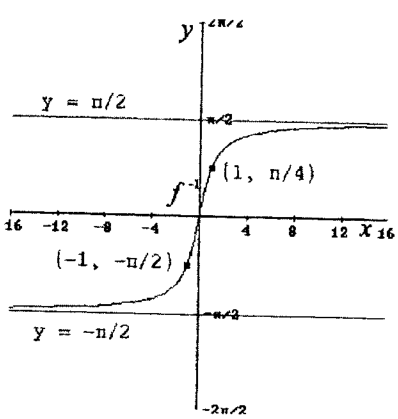
(a) Invertible.



(b) Not invertible; to make it invertible restrict the domain to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.



(c) Invertible.



Problem 2.

(a) Given $y = 2 - \frac{x+1}{x}$, interchange x and y to obtain $x = 2 - \frac{y+1}{y} \Leftrightarrow xy = 2y - (y+1) \Leftrightarrow y = \frac{1}{1-x}$. Hence $f^{-1}(x) = \frac{1}{1-x}$.

(b) Given $y = \frac{x^5}{10} + 7$, interchange x and y to obtain $x = \frac{y^5}{10} + 7 \Leftrightarrow 10(x-7) = y^5 \Leftrightarrow y = \sqrt[5]{10x - 70}$. Hence $f^{-1}(x) = \sqrt[5]{10x - 70}$.

Problem 6.

Given $y = \frac{x}{x+3}$, interchange x and y and solve for y. Now $x = \frac{y}{y+3} \Rightarrow xy + 3x = y \Rightarrow y(1-x) = 3x \Rightarrow y = \frac{3x}{1-x}$. Hence $f^{-1}(x) = \frac{3x}{1-x}$. The domain of f^{-1} is $(-\infty, 1) \cup (1, \infty)$.

Problem 7.

Given $y = \frac{2}{3-x}$, interchange x and y and solve for y. Now $x = \frac{2}{3-y} \Rightarrow 3x - xy = 2 \Rightarrow yx = 3x - 2 \Rightarrow y = \frac{3x-2}{x}$. Hence $f^{-1}(x) = \frac{3x-2}{x} = 3 - \frac{2}{x}$. The domain of f^{-1} is $(-\infty, 0) \cup (0, \infty)$.

Problem 8.

Given $y = \sqrt{x+3}$, interchange x and y and solve for y. Now $x = \sqrt{y+3} \Rightarrow x^2 = y + 3 \Rightarrow y = x^2 - 3$. Hence $f^{-1}(x) = x^2 - 3$. The domain of f^{-1} is the range of f, which is $[0, \infty)$.

Problem 11.

$f(x) = x^3 + 2x - 3 \Rightarrow f'(x) = 3x^2 + 2$. As $f'(x) > 0$ for all x, f is increasing and hence 1-to-1. (Note: algebraically finding an inverse for f would require solving a cubic equation, which is not expected of students using this text.)