Section 13.3 Using Logarithms and Exponentiation to Solve Equations

Problem 2.

(a) $\log(10^{2x}) = \log 93 \Rightarrow 2x = \log 93 \Rightarrow x = \frac{\log 93}{2} (\approx 0.984)$.

(b) $\log 10^{3x+2} = \log 1,000,000 \Rightarrow 3x + 2 = 6 \Rightarrow x = \frac{4}{3}$

(c) $x + 1 = \log_2 7 \Rightarrow x = \log_2(7) - 1 (\approx 1.807)$.

(d) $3^{x+x^2} = 3^1 \Rightarrow \log_3(3^{x+x^2}) = \log_3(3^1) \Rightarrow x + x^2 = 1 \Rightarrow x = \frac{-1 \pm \sqrt{5}}{2} (\approx 0.618, -1.618)$.

(e) $\ln(5B^x) = \ln((2C)^{x+1}) \Rightarrow x \ln(5B) = (x+1)\ln(2C) \Rightarrow x(\ln(5B) - \ln(2C)) = \ln(2C) \Rightarrow x = \frac{\ln(2C)}{\ln(5B) - \ln(2C)}$.

(f) $\ln x = 3 \Rightarrow e^{\ln x} = e^3 \Rightarrow x = e^3 (\approx 20.086)$.

(g) $10^{\log_{10} x} = 10^{17} \Rightarrow x = 10^{17}$

(h) $e^{\ln(5x-40)} = e^3 \Rightarrow 5x - 40 = e^3 \Rightarrow x = \frac{e^3 + 40}{5} (\approx 12.017)$.

(i) $10^{\log_{10}(2x^2+4)} = 10^2 \Rightarrow 2x^2 + 4 = 100 \Rightarrow x = \pm 4\sqrt{3} (\approx \pm 6.928)$.

(j) $2^{x/7} = \frac{16}{3} \Rightarrow \log_2(2^{x/7}) = \log_2(\frac{16}{3}) \Rightarrow \frac{x}{7} = \log_2(\frac{16}{3}) \Rightarrow x = 7\log_2(\frac{16}{3}) (\approx 16.905)$.

Problem 5.

(a) $2^{x^2+x} = 3^x \Rightarrow \ln(2^{x^2+x}) = \ln(3^x) \Rightarrow (x^2 + x)\ln 2 = x \ln 3 \Rightarrow (\ln 2)x^2 + (\ln 2 - \ln 3)x = 0 \Rightarrow$
$x(x \ln x + \ln 2 - \ln 3) = 0 \Rightarrow x = 0$ or $x = \frac{\ln 3}{\ln 2} - 1$.

(b) $\log_3(3^{x^2+2x}) = \log_3 1 \Rightarrow x^2 + 2x = 0 \Rightarrow x(x + 2) = 0 \Rightarrow x = 0$ or $x = -2$.

(c) $12 \ln x - 2(\ln 2 + \ln x) = 10 \Rightarrow 10 \ln x = 10 + 2 \ln 2 \Rightarrow \ln x = \frac{10 + 2\ln 2}{10} \Rightarrow x = e^{\frac{10+2\ln 2}{10}}$.

(d) $(e^x + 3)(e^x - 2) = 0 \Rightarrow e^x = -3$ (not possible) or $e^x = 2 \Rightarrow x = \ln 2$.

(e) $e^{2x} + 8 = 6e^x \Rightarrow e^{2x} - 6e^x + 8 = 0 \Rightarrow (e^x - 2)(e^x - 4) = 0 \Rightarrow e^x = 4$ or $e^x = 2 \Rightarrow x = \ln 4$ or $x = \ln 2$.

(f) $(\ln x)(\ln 5) = \ln 4 + \ln x \Rightarrow (\ln x)(\ln 5 - 1) = \ln 4 \Rightarrow \ln x = \frac{\ln 4}{\ln 5 - 1} \Rightarrow x = e^{\frac{\ln 4}{\ln 5 - 1}}$.

Problem 9.

$\ln(\sqrt{\pi} 3^{1+2x}) = \ln(5^x) \Rightarrow \ln(\sqrt{\pi}) + \ln(3^{1+2x}) = x \ln(5) \Rightarrow \ln(\sqrt{\pi}) + (1 + 2x)\ln(3) = x \ln(5) \Rightarrow \ln(\sqrt{\pi}) +$
$\ln(3) = x(\ln(5) - 2\ln(3)) \Rightarrow x = \frac{\ln(\sqrt{\pi}) + \ln(3)}{\ln(5) - 2\ln(3)} (\approx -2.843)$.

Problem 11.

$\ln(e^{2+x}) = \ln(\pi^{3x+3}) \Rightarrow 2 + x = (3x + 3)\ln(\pi) \Rightarrow 2 - 3\ln(\pi) = 3x(\ln(\pi)) - x \Rightarrow 2 - 3\ln(\pi) = x(3\ln(\pi) - 1)$
$\Rightarrow x = \frac{2 - 3\ln(\pi)}{3\ln(\pi) - 1} (\approx -0.589)$.

Problem 13.

$7 + \pi 3^{x+2} = 6\pi \Rightarrow 3^{x+2} = \frac{6\pi-7}{\pi} \Rightarrow \ln(3^{x+2}) = \ln(\frac{6\pi-7}{\pi}) \Rightarrow (x+2)\ln(3) = \ln(6\pi-7) - \ln(\pi) \Rightarrow$
$x + 2 = \frac{\ln(6\pi-7)-\ln(\pi)}{\ln(3)} \Rightarrow x = \frac{\ln(6\pi-7)-\ln(\pi)}{\ln(3)} - 2 (\approx -0.792).$

Problem 15.

$\ln x^2 = 3 + \ln x \Rightarrow 2\ln x = 3 + \ln x \Rightarrow \ln x = 3 \Rightarrow x = e^3 (\approx 20.086).$

Problem 17.

$[\ln(2x+3)]^2 = 9 \Rightarrow \ln(2x+3) = 3$ or $\ln(2x+3) = -3 \Rightarrow 2x+3 = e^3$ or $2x+3 = e^{-3} \Rightarrow x = \frac{e^3-3}{2} (\approx 8.543)$
or $x = \frac{e^{-3}-3}{2} (\approx -1.475).$

Problem 19.

$e^x(e^x - 5) = 0 \Rightarrow e^x = 0$ (not possible) or $e^x - 5 = 0 \Rightarrow e^x = 5 \Rightarrow x = \ln 5 (\approx 1.609).$

Problem 21.

$e^{2x} - 4e^x + 3 = 0 \Rightarrow (e^x - 3)(e^x - 1) = 0 \Rightarrow e^x = 3$ or $e^x = 1 \Rightarrow x = \ln 3 (\approx 1.099)$ or $x = \ln(1) = 0.$

Problem 23.

$e^{-2x} - e^{-x} = 6 \Rightarrow (e^{-x})^2 - e^{-x} - 6 = 0 \Rightarrow (e^{-x} - 3)(e^{-x} + 2) = 0 \Rightarrow e^{-x} = 3$ or $e^{-x} = -2$ (not possible)
$\Rightarrow x = -\ln(3) = \ln\left(\frac{1}{3}\right) (\approx -1.099).$

Problem 27.

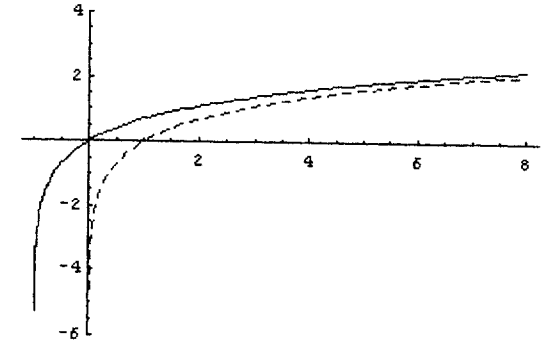
$3^{\ln x} = 5x \Rightarrow \ln(3^{\ln x}) = \ln 5x \Rightarrow (\ln x)(\ln 3) = \ln 5 + \ln x \Rightarrow (\ln x)((\ln 3) - 1) = \ln 5 \Rightarrow \ln x = \frac{\ln 5}{(\ln 3)-1} \Rightarrow$
$x = e^{\frac{\ln 5}{(\ln 3)-1}} (\approx 1.225 \times 10^7).$

Problem 32.

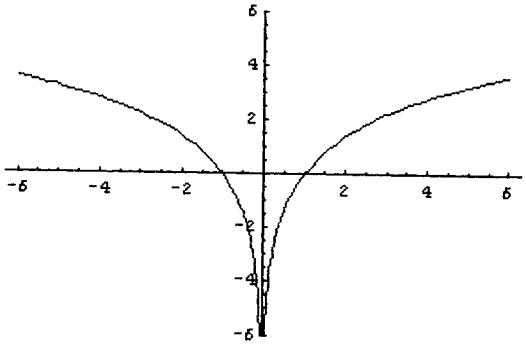
$\ln(x-3) - \ln(2x+1) = 1 \Rightarrow \ln(\frac{x-3}{2x+1}) = 1 \Rightarrow \frac{x-3}{2x+1} = e \Rightarrow x - 3 = e(2x+1) \Rightarrow x(1-2e) = e+3 \Rightarrow$
$x = \frac{e+3}{1-2e} \approx -1.289.$ As values of x must be greater than 3 in the original equation, there is no solution.

Section 13.4 Graphs of Logarithmic Functions: Theme and Variations

Problem 1.



Problem 2.



Problem 3.