

## Section 14.1 The Derivative of Logarithmic Functions

### Problem 1.

$$y = 2 \ln 5x \Rightarrow y = 2(\ln(5) + \ln(x)) \Rightarrow y' = 2\left(\frac{1}{x}\right) = \frac{2}{x}.$$

### Problem 2.

$$y = \pi \ln \sqrt{x} \Rightarrow y = \pi\left(\frac{1}{2}\right) \ln(x) \Rightarrow y' = \left(\frac{\pi}{2}\right)\left(\frac{1}{x}\right) = \frac{\pi}{2x}.$$

### Problem 3.

$$y = \frac{\ln 3x}{5} \Rightarrow y = \frac{1}{5}(\ln(3) + \ln(x)) \Rightarrow y' = \frac{1}{5}\left(\frac{1}{x}\right) = \frac{1}{5x}.$$

### Problem 4.

$$y = x \ln x \Rightarrow y' = (1)(\ln(x)) + (x)\left(\frac{1}{x}\right) = \ln(x) + 1.$$

### Problem 5.

$$y = \frac{\ln \sqrt{2x}}{x} \Rightarrow y = \frac{\frac{1}{2} \ln(2x)}{x} = \frac{\ln(2) + \ln(x)}{2x} \Rightarrow y' = \frac{\left(\frac{1}{x}\right)(2x) - (2)(\ln(2) + \ln(x))}{(2x)^2} = \frac{1 - (\ln(2) + \ln(x))}{2x^2}.$$

### Problem 6.

$$y = 3 \log x \Rightarrow y = 3 \frac{\ln x}{\ln 10}$$
$$y' = \frac{3}{\ln 10} \left(\frac{1}{x}\right) = \frac{3}{x \ln 10}.$$

### Problem 7.

$$y = \frac{\log_2 x}{3} \Rightarrow y = \frac{\ln x}{3 \ln 2} \Rightarrow y' = \frac{1}{3 \ln 2} \left(\frac{1}{x}\right) = \frac{1}{3x \ln 2}.$$

### Problem 8.

$f(x) = \frac{\ln(\sqrt{3x})}{2} + 3 = \frac{\ln(3x)}{4} + 3 = \frac{\ln(3) + \ln(x)}{4} + 3 = \frac{\ln(x)}{4} + \frac{\ln(3)}{4} + 3 \Rightarrow f'(x) = \frac{1}{4}\left(\frac{1}{x}\right) = \frac{1}{4x}$ . We see that  $f'(x) > 0$  on  $(0, \infty)$ , which is the domain of  $f$ . Thus  $f$  is increasing and hence is invertible.

To solve for  $f^{-1}(x)$ , we have:  $x = \frac{\ln(\sqrt{3f^{-1}(x)})}{2} + 3 \Rightarrow 2(x-3) = \ln(\sqrt{3f^{-1}(x)}) \Rightarrow e^{2(x-3)} = \sqrt{3f^{-1}(x)}$   
 $\Rightarrow (e^{2(x-3)})^2 = 3f^{-1}(x) \Rightarrow f^{-1}(x) = \frac{e^{4(x-3)}}{3}$ .

### Problem 10.

$f'(x) = \ln(x) + x\frac{1}{x} = \ln(x) + 1 \Rightarrow f'(x) = 0$  when  $\ln(x) = -1 \Rightarrow x = \frac{1}{e}$ . Now  $f'(x)$  is positive on  $(\frac{1}{e}, \infty)$  and is negative on  $(0, \frac{1}{e})$ . Hence, at its only critical point,  $x = \frac{1}{e}$ ,  $f$  has a local and absolute minimum value of  $f\left(\frac{1}{e}\right) = -\frac{1}{e}$ .

**Problem 1.**

$$y = x^2 \cdot 2^x \Rightarrow \frac{dy}{dx} = (2x)(2^x) + (\ln(2)2^x)(x^2) = x(2^x)(2 + \ln(2)x).$$

**Problem 2.**

$$y = \frac{5 \cdot 2^x}{3} \Rightarrow \frac{dy}{dx} = \frac{5}{3}(\ln(2)2^x).$$

**Problem 3.**

$$y = \frac{x^5 5^x}{5} \Rightarrow \frac{dy}{dx} = \frac{1}{5}((5x^4)(5^x) + (\ln(5)5^x)(x^5)) = \frac{1}{5}(x^4)(5^x)(5 + \ln(5)x).$$

**Problem 4.**

$f(x) = x \cdot 2^x \Rightarrow f'(x) = 2^x + x(2^x \ln 2) = 2^x(1 + x \ln 2)$ . Now  $f'(x) = 0 \Rightarrow 1 + x \ln 2 = 0 \Rightarrow x = -\frac{1}{\ln 2}$  is the only critical point. Moreover, note that  $f'(x) < 0$  for  $x < -\frac{1}{\ln 2}$  and  $f'(x) > 0$  for  $x > -\frac{1}{\ln 2}$ . Hence the absolute minimum value of  $f(x)$  occurs at  $x = -\frac{1}{\ln 2}$  and is  $f(-\frac{1}{\ln 2}) = (-\frac{1}{\ln 2})2^{-\frac{1}{\ln 2}} \approx -0.53$ .

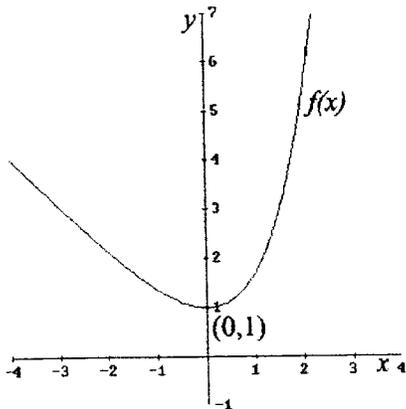
**Problem 5.**

(a)  $f(x) = x^2 + e^x + x^e + e^2 \Rightarrow f'(x) = 2x + e^x + ex^{e-1}$ .

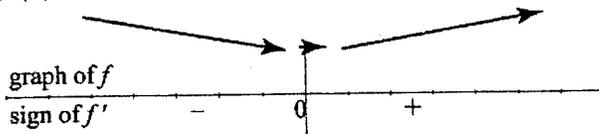
(b)  $f(x) = (\pi - \frac{6}{\sqrt{29}})e^x \Rightarrow f'(x) = (\pi - \frac{6}{\sqrt{29}})e^x$

(c)  $f(x) = (3e^3)e^x \Rightarrow f'(x) = (3e^3)e^x = 3e^{x+3}$

**Problem 8.**

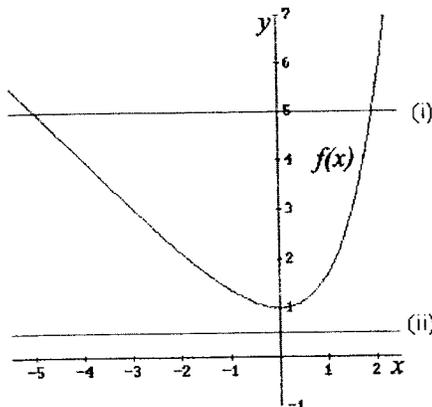


(a)  $f'(x) = e^x - 1$ .  $f'(x) = 0$  when  $x = 1$ .  $f'(x) < 0$  for  $x < 0$  and  $f'(x) > 0$  for  $x > 0$ .



(b) By the first derivative test and part (a),  $x = 0$  is the only local minimum point. The corresponding local minimum value is  $f(0) = e^0 - 0 = 1$ . See the labeled graph above.

(c)



(i) 2 solutions

(ii) No solution