14.3

Problem 6.

$f(x) = x\ln\left(\frac{1}{x}\right) = x\ln(x^{-1}) = -x\ln x \Rightarrow f'(x) = -\left((1)\ln x + \left(\frac{1}{x}\right)(x)\right) = -\ln x - 1.$

Problem 7.

$f(x) = \frac{3\ln(3^6x^7)}{\pi} + \frac{3\ln(3^6)}{\pi} = \frac{3\ln(3^6)+3\ln(x^7)}{\pi} + \frac{3\ln(3^6)}{\pi} = \frac{6\ln(3^6)+21\ln x}{\pi} = \Rightarrow f'(x) = \frac{21}{\pi x}.$

Problem 8.

$f(x) = e^{5x}\ln\left(\frac{\pi}{\sqrt{x}}\right) = e^{5x}(\ln\pi - \ln\sqrt{x}) = e^{5x}\left(\ln\pi - \frac{1}{2}\ln x\right) = (e^{5x})\left(\ln\pi - \frac{1}{2}\ln x\right)$

$\Rightarrow f'(x) = (5e^{5x})\left(\ln\pi - \frac{1}{2}\ln(x)\right) + \left(-\frac{1}{2x}\right)(e^{5x}) = e^{5x}\left(5\left(\ln\pi - \frac{\ln x}{2}\right) - \frac{1}{2x}\right).$

Problem 9.

$f(x) = 3^x(\log x) = 3^x\left(\frac{\ln x}{\ln 10}\right) = \left(\frac{3^x\ln x}{\ln 10}\right) \Rightarrow f'(x) = \frac{((\ln 3)3^x)(\ln x) + (3^x)\left(\frac{1}{x}\right)}{\ln 10} = \frac{3^x(x(\ln 3)(\ln x) + 1)}{x\ln 10}.$

Problem 10.

$f(x) = \frac{\ln(2x^3)}{3e^x} = \frac{\ln 2 + 3\ln x}{3e^x} \Rightarrow f'(x) = \frac{\left(\frac{3}{x}\right)(3e^x) - (\ln 2 + 3\ln x)(3e^x)}{9e^{2x}} = \frac{3 - x\ln 2 - 3x\ln x}{3xe^x}.$

Problem 18.

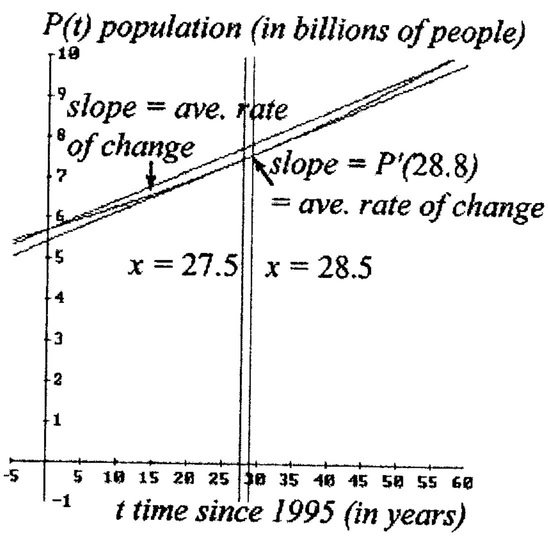
(a) $\frac{9.8 \text{ billion}}{5.7 \text{ billion}} \approx 1.719$. The world's population will increase by 71.9% .

(b) Let $P(t)$ be the total population of the world in billions t years since 1995. As $P(t)$ is exponential, $P(t)$ has the form $P(t) = P_0e^{kt}$, where P_0 and k are constants. Now $P(0) = 5.7 = P_0$, and thus $P(t) = 5.7e^{kt}$. As $P(55) = 9.8$, we have that $P(55) = 9.8 = 5.7e^{55k} \Rightarrow e^{55k} = \frac{9.8}{5.7} \Rightarrow 55k = \ln\left(\frac{9.8}{5.7}\right) \Rightarrow k = \frac{1}{55}\ln\left(\frac{9.8}{5.7}\right) \approx 0.00985$. Therefore, $P(t) = 5.7e^{0.00985t}$. Now $\frac{P(t+1)}{P(t)} = \frac{e^{0.00985(t+1)}}{e^{0.00985t}} = e^{0.00985} \approx 1.0099$. Therefore the annual percentage growth rate is about 0.99%.

(c) $P'(t) = (0.00985)5.7e^{0.00985t} = 0.056145e^{0.00985t} \Rightarrow P'(0) = 0.056145e^{0.00985(0)} = 0.056145$ and $P'(55) = 0.056145e^{0.00985(55)} \approx 0.096513957$. At $t = 0$, the population is growing at a rate of 56.145 million people per year, and at $t = 55$, the population is growing at 96.513957 million people per year.

(d) The average rate of growth is $\frac{P(55)-P(0)}{55-0} = \frac{9.8-5.7}{55} \approx 0.074545454$ billion people per year $= 74,545,455$ people per year.

(e) Solve $\frac{P(55)-P(0)}{55-0} = \frac{4.1}{55} = 0.056145e^{0.00985t}$. Now $e^{0.00985t} = \frac{4.1}{(55)(0.056145)} \Rightarrow 0.00985t = \ln\left(\frac{4.1}{(55)(0.056145)}\right) \Rightarrow t = \left(\frac{1}{0.00985}\right)\ln\left(\frac{4.1}{(55)(0.056145)}\right) \approx 28.8$ years. In 2024, the instantaneous rate of growth and the average rate of growth over the 55-year period are equal.



(f) For North America, $N(t) = N_0(1 + (0.30)\frac{t}{55}) = \frac{0.3N_0t}{55} + N_0$. For Africa, $A(t) = A_0(1 + (3.00)\frac{t}{55}) = \frac{3A_0t}{55} + A_0$.

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Problem 4.

$f(x) = \frac{x}{(x^3+7x)^4} = \frac{x}{(x^2+7)^4x^4} = \frac{1}{(x^2+7)^4x^3} = (x^2+7)^{-4}x^{-3} \Rightarrow$
$f'(x) = \left(-4(x^2+7)^{-5}\left(\frac{d}{dx}(x^2+7)\right)\right)(x^{-3}) + (-3x^{-4})((x^2+7)^{-4}) =$
$(-8x(x^2+7)^{-5})(x^{-3}) + (-3x(x^2+7))^{-4} = -8x^{-2}(x^2+7)^{-5} + (-3x^3-21x)^{-4} = -\frac{8x^2}{(x^2+7)^5} + \frac{1}{(3x^3+21x)^4}.$

Problem 7.

$f(x) = \left(1-\frac{1}{x}\right)e^{-x} \Rightarrow f'(x) = x^{-2}e^{-x} + \left(e^{-x}\left(\frac{d}{dx}(-x)\right)\right)\left(1-\frac{1}{x}\right) =$
$x^{-2}e^{-x} + (-e^{-x})\left(1-\frac{1}{x}\right) = e^{-x}\left(x^{-2}+\frac{1}{x}-1\right) = \frac{e^{-x}(x^2-x-1)}{x^2}.$

Problem 10.

$f(x) = (3x^3+2x)^{13} \Rightarrow f'(x) = 13(3x^3+2x)^{12}\left(\frac{d}{dx}(3x^3+2x)\right) = 13(3x^3+2x)^{12}(9x^2+2).$

Problem 12.

$f(x) = \frac{\pi^2}{3(x^3+2)^6} = \frac{\pi^2}{3}(x^3+2)^{-6} \Rightarrow f'(x) = \frac{\pi^2}{3}\left(-6(x^3+2)^{-7}\left(\frac{d}{dx}(x^3+2)\right)\right) =$
$-2\pi^2(x^3+2)^{-7}(3x^2) = -6\pi^2x^2(x^3+2)^{-7}.$

Problem 16.

$f(x) = \frac{4}{\sqrt{e^x+1}} = 4(e^x+1)^{-1/2} \Rightarrow f'(x) = 4\left(-\frac{1}{2}\right)(e^x+1)^{-3/2}\left(\frac{d}{dx}(e^x+1)\right) =$
$-2e^x(e^x+1)^{-3/2}.$

Problem 19.

$f(x) = \ln(e^x+x^2) \Rightarrow f'(x) = \frac{1}{e^x+x^2}\left(\frac{d}{dx}(e^x+x^2)\right) = \frac{e^x+2x}{e^x+x^2}.$

Problem 20.

$f(x) = x\ln\left(\frac{x}{x^2+1}\right) = x(\ln(x)-\ln(x^2+1)) \Rightarrow$
$f'(x) = (1)(\ln(x)-\ln(x^2+1)) + (x)\left(\frac{1}{x} - \frac{1}{x^2+1}\left(\frac{d}{dx}(x^2+1)\right)\right) = \ln(x)-\ln(x^2+1)+1-\frac{x}{x^2+1}(2x) =$
$\ln(x)-\ln(x^2+1)+1-\frac{2x^2}{x^2+1}.$