

Section 16.2 The Derivative of x^n where n is any Real Number

Problem 1.

$$y' = 2\pi x^{2\pi-1} + 2\ln(\pi)\pi x.$$

Problem 2.

$$y' = \sqrt{3}(2x^2 + 1)^{\sqrt{3}-1}(4x) = 4\sqrt{3}x(2x^2 + 1)^{\sqrt{3}-1}.$$

Problem 3.

$$y' = 3(\sqrt{2} + 1)(3x)^{\sqrt{2}} - \frac{1}{2\sqrt{\pi x^3}}.$$

Problem 4.

$$(a) y' = \pi e^{3t^2 + \pi}(6t) = 6\pi t e^{3t^2 + \pi}.$$

$$(b) y' = \frac{1}{e^t + 1}(e^t) = \frac{e^t}{e^t + 1}.$$

$$(c) y = \frac{\pi^2}{\sqrt{x^2 + 4}} = \pi^2(x^2 + 4)^{-1/2} \Rightarrow y' = -\pi^2 \left(\frac{1}{2}\right)(x^2 + 4)^{-3/2}(2x) = -\pi^2 x(x^2 + 4)^{-3/2} = -\frac{\pi^2 x}{(x^2 + 4)^{3/2}}.$$

$$(d) y = \frac{1}{(\ln x)^{2.6}} = (\ln x)^{-2.6} \Rightarrow y' = -2.6(\ln x)^{-3.6} \left(\frac{1}{x}\right) = -\frac{2.6}{x(\ln(x))^{3.6}}.$$

$$(e) y = \frac{1}{(\ln x^2)^{1.5}} = (2 \ln x)^{-1.5} \Rightarrow y' = -1.5(2 \ln x)^{-2.5} \left(\frac{2}{x}\right) = -\frac{3}{x(2 \ln(x))^{2.5}}.$$

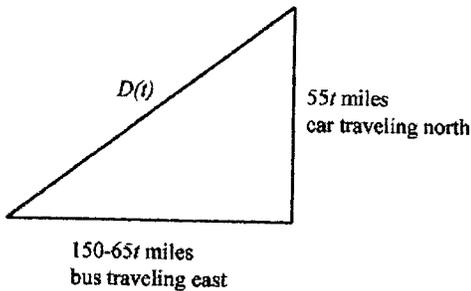
$$(f) y' = \frac{1}{3}(\ln(e^t + 1))^{-2/3} \left(\frac{1}{e^t + 1}(e^t)\right) = \frac{e^t}{(3e^t + 3)(\ln(e^t + 1))^{2/3}}.$$

Section 16.3 Using the Chain Rule

Problem 1.

Let $D(t)$ be the distance between the vehicles at time t . We can just minimize $S(t) = (D(t))^2$ instead of $D(t)$ because $S(t)$ increases as $D(t)$ increases for $D(t) \geq 0$, and hence $S(t)$ is minimized at the same value of t for which $D(t)$ is minimized.

(a) From the Pythagorean Theorem see that $S(t) = (D(t))^2 = (55t)^2 + (150 - 65t)^2$.



Now $S'(t) = 2(55t)(55) + 2(150 - 65t)(-65) = 14,500t - 19,500$, and $S'(t) = 0$ when $t = \frac{39}{29} \approx 1.345$. As $S(t)$ is a quadratic polynomial with positive coefficient of t^2 , $S(t)$ is minimized at $t = \frac{39}{29}$. Therefore the bus and the car will be closest about 1.345 hours after noon.

(b) The minimum distance is $D\left(\frac{39}{29}\right) = \sqrt{\left(55\left(\frac{39}{29}\right)\right)^2 + \left(150 - 65\left(\frac{39}{29}\right)\right)^2} \approx 96.9$ miles.

(c) The bus is $150 - 65\left(\frac{39}{29}\right) \approx 62.6$ miles from the intersection.

Problem 8.

(a) $\left. \frac{dy}{dx} \right|_{x=0} = f'(g(0))g'(0) = f'(2)(-1) = 2.5(-1) = -2.5.$

(b) $y(0) = g(f(0)) = g(3) = 1.$

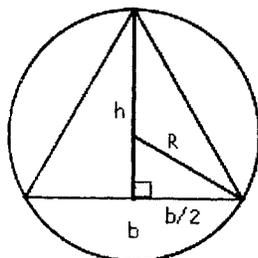
(c) $\left. \frac{d}{dx}(f(x)(g(x))^{-1}) \right|_{x=3} = (f'(3)(g(3))^{-1} - (g(3))^{-2}g'(3)f(3)) = 5(1) - (1)^2(4)(3) = -7.$

(d) $\frac{dy}{dx} = 2f(x)f'(x)g(x^2) + [f(x)]^2g'(x^2)(2x).$ $\left. \frac{dy}{dx} \right|_{x=2} = 2f(2)f'(2)g(4) + [f(2)]^2g'(4)(4) = 2(0.3)(2.5)(3) + (0.3)^2(5)(4) = 6.3.$

(e) $y'(x) = \frac{1}{2}(f(x^2))^{-1/2}f'(x^2)(2x) = \frac{xf'(x^2)}{\sqrt{f(x^2)}}.$ $y'(2) = \frac{4}{\sqrt{11}} = \frac{4}{\sqrt{11}} = \frac{4\sqrt{11}}{11}.$

Problem 31.

Let b be the base and h the height of the inscribed triangle.



From the figure and Pythagorean Theorem, we have that $\frac{1}{2}b = \sqrt{R^2 - (h - R)^2} = \sqrt{2hR - h^2}.$ The area of the triangle is $A(h) = \frac{1}{2}bh = h\sqrt{2hR - h^2}.$ We now maximize $A(h)$ on the interval $[0, 2R].$ Differentiating, we obtain $A'(h) = (1)(\sqrt{2hR - h^2}) + (\frac{1}{2}(2hR - h^2)^{-1/2}(2R - 2h))(h) = \sqrt{2hR - h^2} + (2hR - h^2)^{-1/2}h(R - h)$

$A'(h) = 0$ when:

$$\sqrt{2hR - h^2} = -(2hR - h^2)^{-1/2}h(R - h)$$

$$2hR - h^2 = -h(R - h)$$

$$3hR = 2h^2$$

$$h = \frac{3}{2}R \text{ or } h = 0.$$

To maximize $A(h)$ we compute its value at its critical points $h = 0, \frac{3}{2}R,$ and $2R.$

h	$A(h)$
0	0
$\frac{3}{2}R$	$\frac{3\sqrt{3}}{4}R^2$
$2R$	0

Hence $A(h)$ is maximized when $h = \frac{3}{2}R.$ Now the length of the base is $b =$

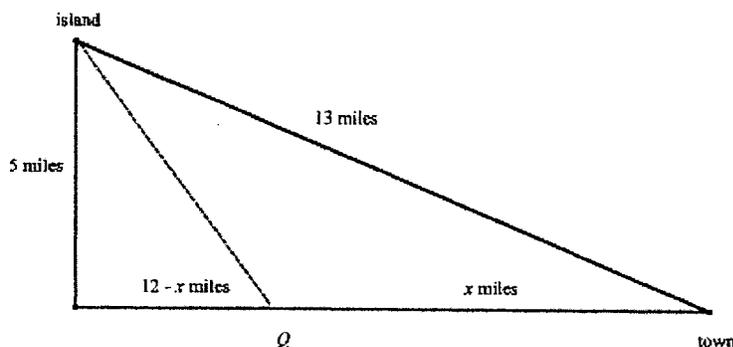
$2\sqrt{3R^2 - \frac{9}{4}R^2} = \sqrt{3}R.$ By the Pythagorean Theorem, we compute the length of one of the two equal-length sides to be

$$L = \sqrt{\frac{b^2}{4} + h^2} = \sqrt{\frac{3R^2}{4} + \frac{9R^2}{4}} = \sqrt{3}R.$$

As all sides of triangle have equal lengths, the triangle is equilateral.

Problem 34.

Let c be the cost in dollars per mile of cable under ground. The cost of the installing the cable under water is $1.6c$ dollars per mile.



From the Pythagorean Theorem and the figure above, we see that the distance from the point P closest to the island on the shore is $\sqrt{13^2 - 5^2} = 12$ miles. Suppose the cable is run from the island to a point Q on shore x miles from town. Also from the Pythagorean Theorem and the figure above, we see that the distance the cable runs underwater is $d_w(x) = \sqrt{5^2 + (12 - x)^2} = \sqrt{x^2 - 24x + 169}$. The total cost of installing the cable is $C(x) = cx + 1.6cd_w(x) = cx + 1.6c\sqrt{x^2 - 24x + 169}$. We now minimize $C(x)$ on $[0, 12]$. Differentiating, we obtain

$$C'(x) = c + 1.6c\left(\frac{1}{2}(x^2 - 24x + 169)^{-\frac{1}{2}}(2x - 24)\right) = c + 1.6c(x^2 - 24x + 169)^{-\frac{1}{2}}(x - 12) = \frac{c(5\sqrt{x^2 - 24x + 169} - 8(x - 12))}{5\sqrt{x^2 - 24x + 169}}$$

Now $C'(x) = 0$ when $5\sqrt{x^2 - 24x + 169} - 8(x - 12) = 0$. We now solve this last equation for x to determine the critical points of $C(x)$ on $(0, 12)$.

$$\begin{aligned} 5\sqrt{x^2 - 24x + 169} &= 8(x - 12) \\ \sqrt{x^2 - 24x + 169} &= 1.6(x - 12) \\ x^2 - 24x + 169 &= 1.6^2(144 - 24x + x^2) \\ 1.56x^2 - 37.44x + 199.64 &= 0 \end{aligned}$$

Using the quadratic formula, we obtain $x = \frac{37.44 \pm \sqrt{(-37.44)^2 - 4(1.56)(199.64)}}{2(1.56)} = 12 \pm \frac{25\sqrt{39}}{39}$. Decimal approximations of these values are $x = 16.0032$ and $x = 7.9968$. As the domain of $C(x)$ is $[0, 12]$, the only critical points in $(0, 12)$ is $x = 25 - \frac{25\sqrt{39}}{39} \approx 7.9968$.

To minimize $C(x)$, we compute $C(x)$ at the endpoints $x = 0$ and $x = 12$ and at the critical point $x = 25 - \frac{25\sqrt{39}}{39} \approx 7.9968$

x	$C(x)$
0	$20.8c$
$25 - \frac{25\sqrt{39}}{39}$	$(12 + \sqrt{39})c \approx 18.245c$
12	$20c$

Therefore, the cable should run $25 - \frac{25\sqrt{39}}{39} \approx 7.9968$ miles along the shore.