**Name:**________________________________

# Math Xa Midterm Examination II—Thursday, November 20, 2003

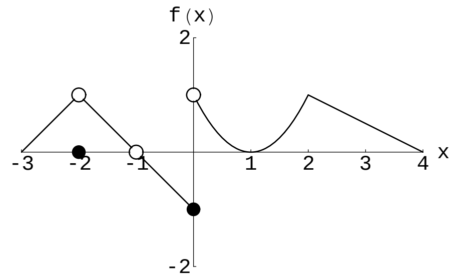
*Please circle your section:*

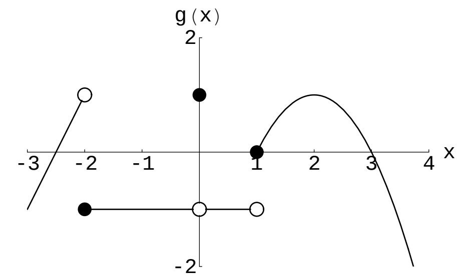
| | | |
|---|---|---|
| Thomas Judson<br>Margaret Barusch (CA)<br>MWF 11–12 | Derek Bruff<br>Connie Zong (CA)<br>MWF 12–1 | Jerrel Mast<br>Jennie Schiffman (CA)<br>MWF 11–12 |
| Maryam Mirzakhani<br>Evan Hepler-Smith (CA)<br>MWF 10–11 | Nicholas Ramsey<br>Maria van Wagenberg (CA)<br>MWF 10–11 | |

| Problem Number | Possible Points | Score |
|---|---|---|
| 1 | 14 | |
| 2 | 4 | |
| 3 | 6 | |
| 4 | 10 | |
| 5 | 12 | |
| 6 | 5 | |
| 7 | 6 | |
| 8 | 6 | |
| 9 | 6 | |
| 10 | 10 | |
| 11 | 6 | |
| 12 | 5 | |
| 13 | 10 | |
| Total | 100 | |

**Directions—Please Read Carefully!** You have two hours to take this midterm. Make sure to use correct mathematical notation. Any answer in decimal form must be accurate to three decimal places, unless otherwise specified. To receive full credit on a problem, you will need to justify your answers carefully—unsubstantiated answers will receive little or no credit (except if the directions for that question specifically say no justification is necessary, such as in a True/False section). Please be sure to write neatly—illegible answers will receive little or no credit. If more space is needed, use the back of the previous page to continue your work. Be sure to make a note of this on the problem page so that the grader knows where to find your answers. You may use a calculator on this exam, but no other aids are allowed. **Good Luck!!!**

1. (14 points) Let $f$ and $g$ be the functions whose graphs are given below. Use the graphs to evaluate the following expressions. If the expression cannot be evaluated, say why.




(a) $\lim_{x \to -2} f(x)$

(b) $f(-2)$

(c) $\lim_{x \to 1^+} g(x)$

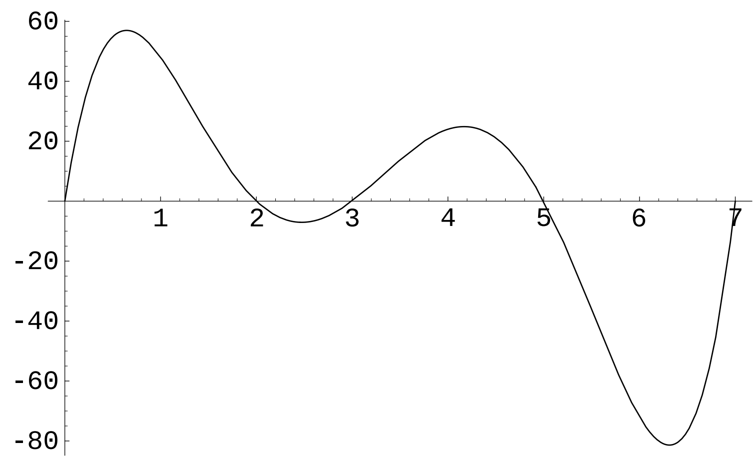
(d) $\lim_{x \to -1} [2f(x) - 3g(x)]$

(e) $\lim_{x \to 1^-} f(x)g(x)$

(f) $\lim_{x \to 1^+} f(x)g(x)$

(g) $\lim_{x \to 1} f(x)g(x)$

2. (4 points) The graph of the derivative $f'$ of a function $f$ is shown below.




(a) On what intervals is $f$ increasing?

(b) At what $x$-values does $f$ have a local max?

3. (6 points) Find the derivative of

$$f(x) = \frac{2}{x}$$

using the definition of the derivative.

*Show all work to receive full credit. You must calculate a difference quotient and take a limit. You will receive no credit for simply applying the power rule.*

4. (10 points) An object is thrown straight up into the air. It height above the ground as function of time is given by

$$h(t) = -16t^2 + 10t + 6,$$

where $h$ is measured in feet and $t$ is measured in seconds, and $t = 0$ corresponds to the time the object was thrown.

(a) What is the initial height of the object?

(b) How high does the object rise?

(c) When does the object hit the ground?

(d) What is the velocity of the object when it hits the ground?

(e) What is the average velocity of the object between the time it is thrown and the time it hits the ground?

5. (12 points) Suppose that $f(4) = 4$, $f'(4) = -3$, $g(4) = 2$, and $g'(4) = -1$. Find each of the following values.

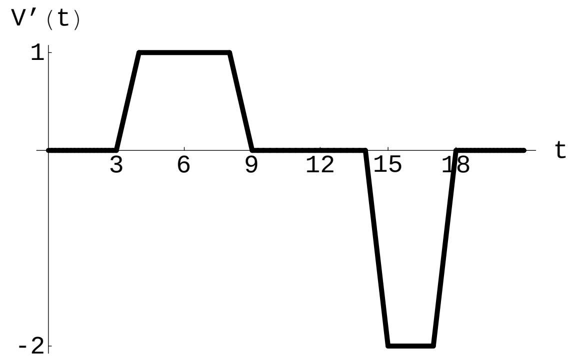
(a) $h'(4)$, if $h(x) = x^2 f(x) + 3g(x)$

(b) $h'(4)$, if $h(x) = \sqrt{x} f(x) - e^x g(x)$

(c) $h'(4)$, if $h(x) = f(x)/g(x)$

(d) Find an equation of a line tangent to $g$ at $x = 4$.

6. (5 points) A child inflates a balloon, admires it for a while and then lets the air out of the balloon at a constant rate. If $V(t)$ gives the volume of the balloon at time $t$, in seconds, the figure below shows $V'(t)$ as a function of $t$.



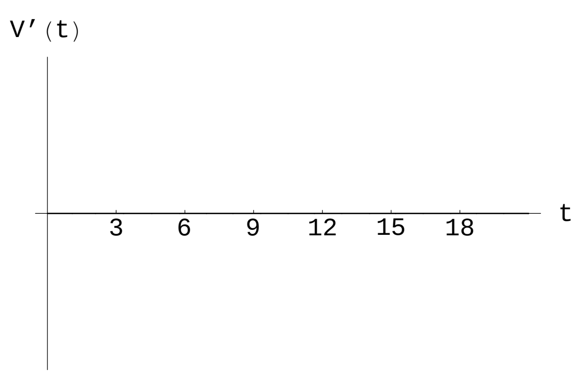

At what time does the child:

(a) Begin to inflate the balloon?

(b) Finishing inflating the balloon?

(c) Begin to let the air out of the balloon?

(d) What would the graph of $V'(t)$ look like if the child had alternated between pinching and releasing the open end of the balloon, instead of letting the air out at a constant rate?



7. (6 points) Estimate $\sqrt[3]{8.1}$ using linear approximation. *Hint*: If $f(x) = \sqrt[3]{x}$, then

$$f'(x) = \frac{1}{3(\sqrt[3]{x}\,)^2}.$$

8. (6 points) Consider the function $f(x) = x^3$ on the interval $[0, 2]$.

(a) Compute the average rate of change of $f$ on the interval.

(b) Is there a point (or points) in the interval at which the slope of the tangent line is equal to this average rate of change? If so, what is it (are they)?

9. (6 points) The graph of $f(t)$, given below, tells us the position of a particle at time $t$. List the following quantities in order, *smallest to largest*.

(a) The average velocity between $t = 1$ and $t = 3$.

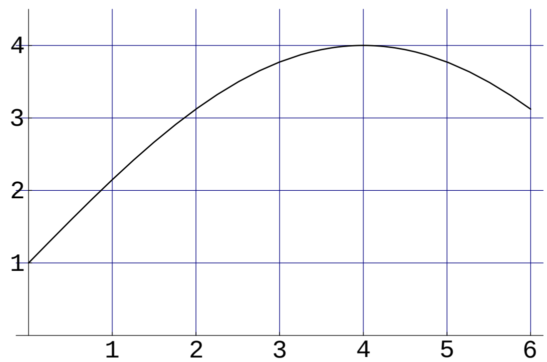
(b) The average velocity between $t = 5$ and $t = 6$.

(c) The instantaneous velocity at $t = 1$.

(d) The instantaneous velocity at $t = 3$.

(e) The instantaneous velocity at $t = 5$.

(f) The instantaneous velocity at $t = 6$.



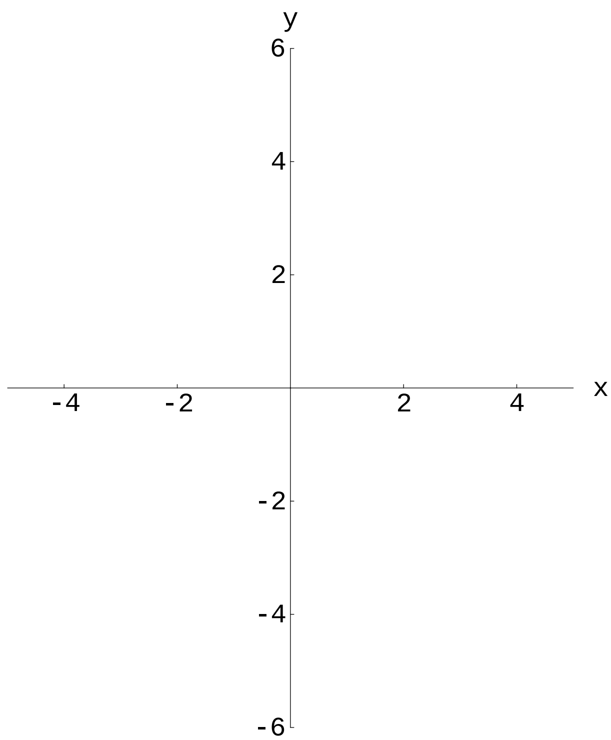
10. (10 points) Carbon-14 or $C_{14}$ is a radioactive isotope of carbon with a half-life of 5730 years. We can use Carbon-14 to date organic remains.

   (a) Let $C_0$ denote the amount of $C_{14}$ in a living organism. Find a formula for $C(t)$, the amount of $C_{14}$ in this organism, $t$ years after it has died.

   (b) A picture supposedly painted by Vermeer (1632–1675) contains 99.5% of its $C_{14}$. From this information, decide whether or not the picture is a forgery. Support your answer.

11. (6 points) Assume that $f$ is a continuous function on the closed interval $[-3, 3]$ with $f(-3) = 4$ and $f(3) = 1$. Also, assume that $f'$ and $f''$ exist and are continuous on $(-3, 3)$. Use the information in the table below to sketch a possible graph of $f$.

| $x$ | $-3 \le x < -1$ | $-1$ | $-1 < x < 0$ | $0$ | $0 < x < 1$ | $1$ | $1 < x \le 3$ |
|---|---|---|---|---|---|---|---|
| $f'(x)$ | $+$ | $0$ | $-$ | $-$ | $-$ | $0$ | $-$ |
| $f''(x)$ | $-$ | $-$ | $-$ | $0$ | $+$ | $0$ | $-$ |



12. (5 points) Sketch the graph of a *single* function $f$ which satisfies *all* of the following conditions.

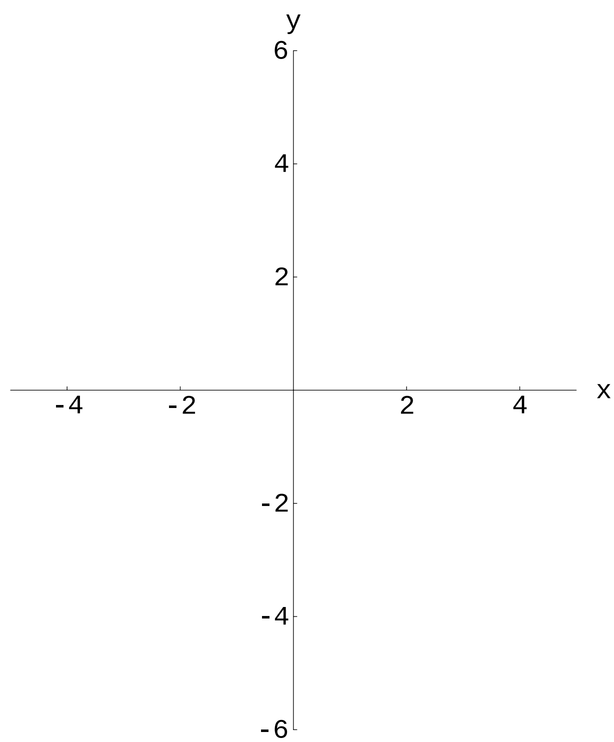
(a) $\lim_{x\to 0} f(x) = \infty$.

(b) $\lim_{x\to -2} f(x) = 0$.

(c) $f(-2)$ undefined

(d) $\lim_{x\to 2^-} f(x) = 1$.

(e) $\lim_{x\to 2} f(x)$ does not exist.



13. (10 points) Mark each statement as *True* or *False* based on the graph of the function $f$ shown below, by circling T or F.

   (a) **T** **F** $f$ has a removable discontinuity at $x = -2$. In other words, can the function be made continuous by defining or redefining the function.

   (b) **T** **F** $f$ has a removable discontinuity at $x = 0$.

   (c) **T** **F** $f$ has no discontinuities in the interval $(-2, 0)$.

   (d) **T** **F** $f$ is differentiable at $x = 1$.

   (e) **T** **F** $f$ is not differentiable at $x = 3$.