Math Xa
Worksheet—Concavity and the Second Derivative

Fall 2003

1. Let $f(x) = e^x/x$.

   (a) Find all the critical points of $f(x)$.

   (b) Identify all local extrema.

   (c) Does f have an absolute maximum value? If so, where is it attained? What is its value?

(d) Does f have an absolute minimum value? If so, where is it attained? What is its value?

(e) Answer parts (c) and (d) if x is restricted to $(0, \infty)$.

2. Find all local extrema of $f(x) = \frac{1}{5}x^5 - x^4 + \frac{4}{3}x^3 + 2$. Also determine the concavity and inflection points of f.

3. Suppose that f is a continuous function such that $f(3) = 2$, $f'(3) = 0$, and $f''(3) = 3$. At $x = 3$, does f have a local maximum, a local minimum, neither a local maximum nor a local minimum, or is it impossible to determine? Explain your answer.