# Math Xa
# Worksheet—More Exponential Functions
# Solutions

Fall 2003

1. Evaluate the derivative of each of the following functions.

   (a) $y = xe^{10x}$

   **Solution.** Use the product rule,

   $$y' = \left(\frac{d}{dx}x\right)e^{10x} + x\left(\frac{d}{dx}e^{10x}\right) = e^{10x} + 10xe^{10x}.$$

   (b) $f(t) = e^t + e^\pi + \pi^e$

   **Solution.** Remember that $e^\pi$ and $\pi^e$ are both constant terms and their derivative is zero. Thus,

   $$f'(t) = e^t.$$

   (c) $x(t) = \dfrac{t^3 - 2t + 4}{e^{3t} + 2}$

   **Solution.** Use the quotient rule,

   $$\begin{aligned} x'(t) &= \frac{\left[\frac{d}{dt}(t^3 - 2t + 4)\right](e^{3t} + 2) - (t^3 - 2t + 4)\frac{d}{dt}(e^{3t} + 2)}{(e^{3t} + 2)^2} \\ &= \frac{(3t^2 - 2)(e^{3t} + 2) - 3e^{3t}(t^3 - 2t + 4)}{(e^{3t} + 2)^2} \end{aligned}$$

(d) $y = \dfrac{e^x - e^{-x}}{2}$

**Solution.** You can use the quotient rule to solve this problem, but it is easier if you write the function as

$$y = \frac{1}{2}e^x - \frac{1}{2}e^{-x}.$$

In this case,

$$y' = \frac{1}{2}e^x + \frac{1}{2}e^{-x}.$$

(e) $f(x) = (x^2 - 2x + 5)e^x$

**Solution.** Using the product rule, we get

$$f(x) = (2x - 2)e^x + (x^2 - 2x + 5)e^x.$$

2. According to a report from the General Accounting Office, during the 14-year period between the school year 1980–1981 and the school year 1994–1995 the average tuition at four-year public colleges increased by 234%. During the same period, average household income increased by 82%, and the Labor Department's Consumer Price Index (CPI) increased by 74%.[1]

   (a) Assuming exponential growth, determine the annual percentage increase for each of these three measures.

   **Solution.** Tuition, average household income, and the CPI can be modeled by

   $$f(t) = C_0 b^t,$$

   where $b - 1$ is the annual growth rate and $C_0$ is the amount at time $t = 0$. Thus, if tuition is going up at a rate of 9% each year, you would pay $C_0(1.09)^{10}$ dollars in ten years, where $C_0$ is the amount that you are paying now. We can now solve our problem. Assume that $C_0 = 1$ or 100%.

   - For tuition, $b^{14} = 3.34$. Thus, $b = (3.34)^{1/14} \approx 1.0626$ or 6.26%.

[1] *Boston Globe*, August 16, 1996.

- For household income, $b^{14} = 1.82$. Thus, $b = (1.82)^{1/14} \approx 1.0900$ or 9.00%.
- For the CPI, $b^{14} = 1.74$. Thus, $b = (1.74)^{1/14} \approx 1.0404$ or 4.04%.

(b) The average cost of tuition in 1994–1995 was \$2865 for in-state students. What was it in 1980–1981?

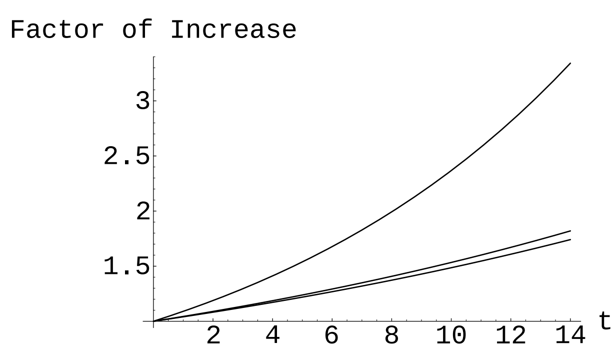
**Solution.** Let $T(t)$ denote the in-state tuition function $t$ years after the 1980–1981 school year. Then

$$2865 = T(14) = T_0 \cdot 3.34.$$

Hence, $T_0 = 857.78$. The average cost of tuition for the 1980–1981 school year was \$857.78.

(c) Starting with an initial value of one unit for each of the three quantities, average tuition at four-year public colleges, average household income, and the CPI, sketch on a single set of axes the graphs of the three functions over this 14 year period.

**Solution.**




(d) Suppose that a family has two children born 14 years apart. In 1980–1981, the tuition cost of sending the elder child to college represented 15% of the family's total income. Assuming that their income has increased at the same pace as the average household, what percent of the income was needed to send the younger child to college in 1994–1995?

**Solution.** Let $I(t)$ denote the family's income $t$ years after the 1980–1981 school year and

$$I_0 = I(0) = \frac{T_0}{0.15}.$$

We then have

$$I(t) = I_0(1.82)^{t/14}.$$

If $T(14) = I(14)r$, then

$$1.82 I_0 r = 3.34 T_0 = 3.34(0.15 I_0)$$

and so $1.82r = 3.34(0.15)$ or $r \approx 0.2753$. Therefore, for the 1994–1995 school year, the family will need about 27.53% of their income to send the younger child to college.