Math Xa
Worksheet—Differentiating Exponential and Logarithmic Functions
Solutions

Fall 2003

1. Find the derivative of each of the following functions.

   (a) $y = x^e + \pi^x + e^\pi$

   Solution.
$$y' = ex^{e-1} + \pi^x(\ln \pi).$$

   (b) $y = x \log_3 x$

   Solution. Using the product rule,
$$y' = x\frac{1}{x \ln 3} + \log_3 x.$$

   (c) $y = \frac{1}{2} \ln x$

   Solution.
$$y' = \frac{1}{2x}.$$

   (d) $y = \ln \sqrt{x}$

   Solution. Since
$$y = \ln \sqrt{x} = \frac{1}{2} \ln x,$$
   the derivative of y is
$$y' = \frac{1}{2x}.$$

(e) $y = \ln(x\sqrt{x}\,)$

Solution. Using the properties of logarithms,

$$y = \ln(x\sqrt{x}\,) = \frac{3}{2}\ln x.$$

Therefore,

$$y' = \frac{3}{2x}.$$

(f) $y = \ln\left(\frac{x^2\sqrt{x}}{(x-2)(x+5)}\right)$

Solution. Using the properties of logarithms,

$$y = \ln\left(\frac{x^2\sqrt{x}}{(x-2)(x+5)}\right) = \frac{5}{2}\ln x - \ln(x-2) - \ln(x+5).$$

Therefore,

$$y' = \frac{5}{2x} - \frac{1}{x-2} - \frac{1}{x+5}.$$

2. In early summer the fly population of Maine grows exponentially. The population at any time t (measured in days) can be given by $P = P_0e^{kt}$ for some constant k. Suppose at some date, which we will designate as $t = 0$, there are 200 flies. Thirty days later, there are 900 flies.

(a) Find P_0 and k.

Solution. First,

$$P_0 = P_0e^{k\cdot 0} = P(0) = 200.$$

Since

$$900 = P(30) = 200e^{30k},$$

we know that

$$\frac{9}{2} = e^{30k},$$

or

$$k = \frac{1}{30}\ln\left(\frac{9}{2}\right) \approx 0.0501.$$

Therefore,

$$P(t) = 200e^{0.0501t}.$$

(b) The mosquito population is also growing exponentially. At time $t = 0$ there are 100 mosquitos, and the mosquito population doubles every 10 days. Write a function $M(t)$ that gives the number of mosquitos at time t.

Solution. Since the population grows exponentially,

$$M(t) = M_0 e^{kt}.$$

Since the initial population is 100,

$$M_0 = M_0 e^{k \cdot 0} = M(0) = 100.$$

Since the doubling time for the population is 10 days,

$$200 = M(10) = 100e^{10k}.$$

Thus,

$$k = \frac{1}{10} \ln 2 \approx 0.0693.$$

Therefore,

$$M(t) = 100e^{0.0693t}.$$

(c) When will the mosquito and fly populations be equal?

Solution. The two populations will be equal when

$$200e^{0.0501t} = 100e^{0.0693t}$$

or at

$$t = \frac{1}{0.0192} \ln 2 \approx 36 \text{ days}.$$

(d) Find $P'(t)$.

Solution.

$$P'(t) = 10.02e^{0.0501t},$$

(e) Find $M'(t)$.

Solution.

$$M'(t) = 6.93e^{0.0693t}.$$

(f) Find the rate at which each population is growing when the two populations are equal. Which population is growing more rapidly?

Solution. Since $P'(36) = 60.836$ and $M'(36) = 83.987$, the mosquito population is growing more rapidly, at a rate of approximately 84 mosquitos per day.