Math Xa Fall 2003

Lab 5: Derivatives

October 28, 2003

Name _____

Section Instructor _____

Collaborators _____

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1. Sketch the graph of a single function f that satisfies all of the following conditions.

   (a) $f(x) > 0$ for x in $(-2, 6)$

   (b) $f(x) < 0$ for x in $(-\infty, -2) \cup (6, \infty)$

   (c) $f(x) = 0$ for $x = -2, 6$

   (d) $f'(x) > 0$ for x in $(-\infty, -1) \cup (1, 4)$

   (e) $f'(x) < 0$ for x in $(4, \infty)$

   (f) $f'(x) = 0$ for x in $[-1, 1]$ and for $x = 4$

   (g) $f''(x) > 0$ for x in $(-\infty, -3) \cup (1, 2)$

   (h) $f''(x) < 0$ for x in $(-3, -1) \cup (2, \infty)$

2. The height in feet of a ball t seconds after it is thrown is given by $h(t) = -16t^2 + 32t + 48$.

   (a) Find the time t_m at which the ball reaches its maximum height.

   (b) Find the average velocity of the ball over the time interval $[0, t_m]$.

   (c) At what t value in the interval $[0, t_m]$ does the ball have an instantaneous velocity equal to the average velocity you found in part (c)?

3. If the tangent line to the curve $y = f(x)$ at the point $(5, 1)$ also passes through the point $(2, 3)$, then what is $f'(5)$?

4. Let $f(x) = x^3 - x + 1$.

   (a) Find $f'(a)$ using the definition of the derivative $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$.

   (b) Find $f'(a)$ using the definition of the derivative $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$.

   (c) Which of the two methods do you prefer and why? Please answer in complete sentences.