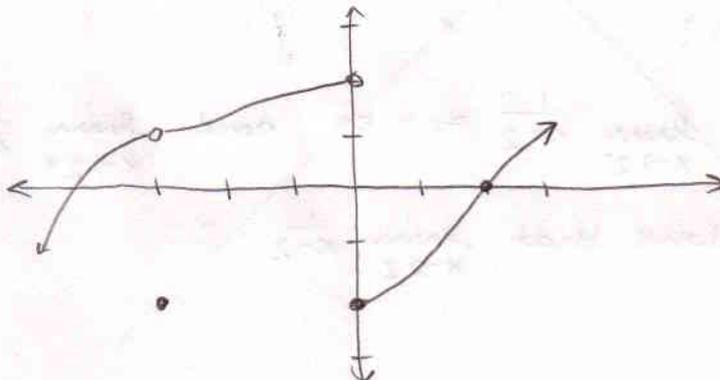


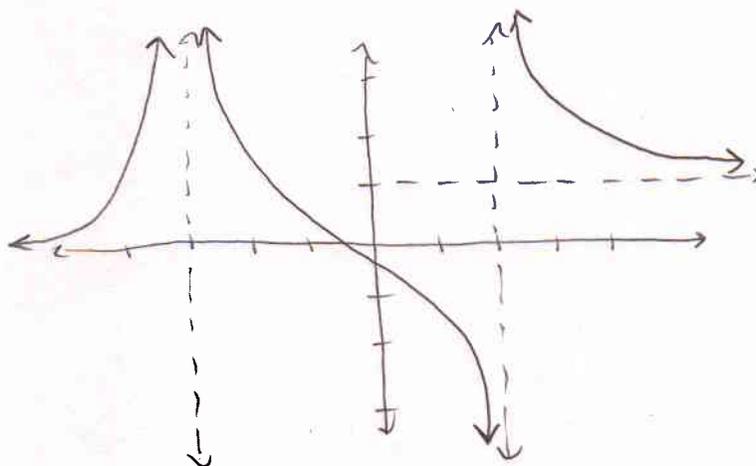
1. Sketch the graph of a single function  $f$  that satisfies all of the following conditions.

- (a)  $\lim_{x \rightarrow -3} f(x) = 1$
- (b)  $f(-3) = -2$
- (c)  $\lim_{x \rightarrow 0^-} f(x) = 2$
- (d)  $\lim_{x \rightarrow 0^+} f(x) = -2$
- (e)  $f(0) = -2$
- (f)  $\lim_{x \rightarrow 2} f(x) = 0$
- (g)  $f(2) = 0$



2. Sketch the graph of a single function  $f$  that satisfies all of the following conditions.

- (a)  $\lim_{x \rightarrow -3} f(x) = \infty$
- (b)  $\lim_{x \rightarrow 2^-} f(x) = -\infty$
- (c)  $\lim_{x \rightarrow 2^+} f(x) = \infty$
- (d)  $\lim_{x \rightarrow -\infty} f(x) = 0$
- (e)  $\lim_{x \rightarrow \infty} f(x) = 1$



3. Identify the discontinuities of the following functions. For each discontinuity you find, tell *where* it is, *why* it is a discontinuity, and *whether* or not the discontinuity is removable.

Your answers to the *why* question should be one of the following: (1)  $f(a)$  is not defined, (2)  $\lim_{x \rightarrow a} f(x)$  does not exist, or (3)  $\lim_{x \rightarrow a} f(x) \neq f(a)$ .

$$(a) f(x) = \frac{1}{x-2}$$

Where?  $x = 2$

Why?  $f(2)$  is not defined or  $\lim_{x \rightarrow 2} f(x)$  does not exist

Removable?

Since  $\lim_{x \rightarrow 2^-} \frac{1}{x-2} = -\infty$ , it follows that

$\lim_{x \rightarrow 2} \frac{1}{x-2}$  does not exist. Thus the

discontinuity is not removable.

$$(b) f(x) = \frac{3-\sqrt{x}}{x-9}$$

Where?  $x = 9$

Why?  $f(9)$  is not defined or  $\lim_{x \rightarrow 9} f(x) \neq f(9)$

Removable?

$$\begin{aligned} \lim_{x \rightarrow 9} \frac{3-\sqrt{x}}{x-9} &= \lim_{x \rightarrow 9} \frac{3-\sqrt{x}}{x-9} \cdot \frac{3+\sqrt{x}}{3+\sqrt{x}} = \lim_{x \rightarrow 9} \frac{9-x}{(x-9)(3+\sqrt{x})} \\ &= \lim_{x \rightarrow 9} \frac{-(x-9)}{(x-9)(3+\sqrt{x})} = \lim_{x \rightarrow 9} \frac{-1}{3+\sqrt{x}} = \frac{-1}{3+3} = -\frac{1}{6} \end{aligned}$$

Since  $\lim_{x \rightarrow 9} \frac{3-\sqrt{x}}{x-9}$  exists, the discontinuity is

removable.

$$(c) f(x) = \frac{x-7}{|x-7|}$$

Where?  $x = 7$

Why?  $f(7)$  is not defined or  $\lim_{x \rightarrow 7} f(x)$  does not exist

Removable?

$$\lim_{x \rightarrow 7^-} \frac{x-7}{|x-7|} = \lim_{x \rightarrow 7^-} \frac{x-7}{-(x-7)} = \lim_{x \rightarrow 7^-} -1 = -1$$

$$\lim_{x \rightarrow 7^+} \frac{x-7}{|x-7|} = \lim_{x \rightarrow 7^+} \frac{x-7}{x-7} = \lim_{x \rightarrow 7^+} 1 = 1$$

Since the left hand limit  $\neq$  right hand limit,

$\lim_{x \rightarrow 7} \frac{x-7}{|x-7|}$  does not exist.  $\Rightarrow$  NOT removable