Math Xa Fall 2003 Name SOLUTIONS

Lab 9: Modeling with Logarithmic Functions Section Instructor

December 4, 2003 Collaborators

1. The geologist C. F. Richter defined the magnitude of an earthquake to be $\log \frac{I}{S}$, where I is the intensity of the quake (measured by the amplitude of a seismograph 100 km away from the epicenter) and S is the intensity of a "standard" earthquake (where the amplitude is only 1 micron $= 10^{-4}$ cm). *Earthquake data given below were taken from the U. S. Geological Survey web site,* `http://earthquake.usgs.gov/`.

   (a) The 1989 Major League Baseball World Series was interrupted by an earthquake centered close to San Francisco, California, the location of the third game of the series. The earthquake measured 6.9 on the Richter scale. San Francisco was also rocked by a major earthquake in 1906, this one measuring 7.8 on the Richter scale. How many times more intense was the 1906 quake than the 1989 quake?

I_1 = intensity of 1906 quake

I_2 = intensity of 1989 quake

$7.8 = \log \frac{I_1}{S}$ $\quad$ $6.9 = \log \frac{I_2}{S}$

$10^{7.8} = \frac{I_1}{S}$ $\quad$ $10^{6.9} = \frac{I_2}{S}$

$I_1 = S \cdot 10^{7.8}$ $\quad$ $I_2 = S \cdot 10^{6.9}$

$$\frac{I_1}{I_2} = \frac{S \cdot 10^{7.8}}{S \cdot 10^{6.9}} = 10^{7.8-6.9}$$

$$= 10^{.9} \approx \boxed{7.94 \text{ times as intense}}$$

   (b) The largest earthquake recorded since 1900 occurred in Chile in 1960. Its magnitude was 9.5 on the Richter scale. How many times more intense was the 1960 Chile quake than the 1906 San Francisco quake?

I_3 = intensity of Chile quake

$I_3 = S \cdot 10^{9.5}$

$$\frac{I_3}{I_1} = \frac{S \cdot 10^{9.5}}{S \cdot 10^{7.9}} = 10^{1.6}$$

$$\approx \boxed{39.8 \text{ times as intense}}$$

   (c) Find a formula for the intensity of an earthquake in terms of its magnitude. (Your formula may include the constant S.)

$M = \log \frac{I}{S}$

$10^M = \frac{I}{S}$

$I = S \cdot 10^M$

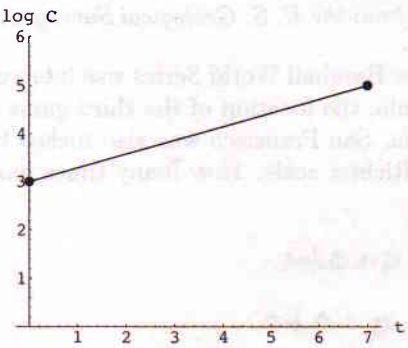
$$\boxed{I(M) = S \cdot 10^M}$$

(d) What effect does a 1 point increase on the Richter scale have on the intensity of an earthquake?

$I(M+1) = 5 \cdot 10^{M+1} = 10 \cdot 5 \cdot 10^{M} = 10 \cdot I(s)$

$\Rightarrow$ increases intensity by a factor of 10

2. An environmental policy advisory board is studying information supplied to them concerning pollution levels in one of India's rivers. The data have been presented as follows.



On the vertical axis is the log of the number of coliform organisms per 100 mL of water water, and on the horizontal axis is time, measured in years from the date the study of the river began. The data look more or less like a line passing through the points (0, 3) and (7, 5). This kind of presentation, plotting the logarithm of some quantity instead of plotting the quantity itself, is a **semilog plot**.

(a) Find a formula for $C(t)$, the number of coliform organisms per 100 mL of water as a function of time.

Equation of the line $y = \log C(t)$

slope $= \frac{5-3}{7-0} = 2/7$

y-intercept $= 3$

$y = \frac{2}{7}t + 3$

$\Rightarrow \log C(t) = \frac{2}{7}t + 3$

$\boxed{C(t) = 10^{\frac{2}{7}t+3}}$

(b) Show that if a quantity grows according to the equation $y = Ce^{kt}$, then $\ln y$ is a linear function of t. (Thus, if a quantity grows exponentially, its semilog plot will be a straight line.) What is the significance of the slope of this line? The vertical intercept of the line?

$y = Ce^{kt}$

$\ln y = \ln Ce^{kt}$

$= \ln C + \ln e^{kt}$

$= \ln C + kt \ln e$

$= \ln C + kt$

slope $= k$: The slope of the line gives an indication of how fast the quantity is growing.

vertical intercept $= \ln C$: This is the natural log of the initial quantity.