Solutions to the First Exam

#1. (a) $f'(1) \approx \dfrac{f(1+h)-f(1)}{h}$ for h small. So, to numerically approximate $f'(1)$ choose a small h.

$$f'(1) \approx \frac{f(1.001)-f(1)}{.001} = \frac{\dfrac{-3(1.001)+1}{1.001+5} - \dfrac{-3(1)+1}{1+5}}{.001} = \frac{\dfrac{-2.003}{6.001}+\dfrac{2}{6}}{.001} \approx -.444$$

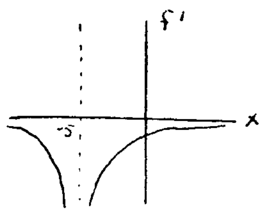
Common errors: ① Some people divided by 1.001. Remember that $f'(x) \approx \frac{\Delta y}{\Delta x}$ and Δx is only .001

② Some folks are rounding off too soon: e.g. $f(1.001) \approx -.3337777037\ldots$ If you write $-.334$ then $\frac{-.334-(-.333)}{.001} = -1$ and you're far off. Recall that dividing by .001 is multiplying by 1000. The best idea is to let your calculator work for you & don't round off until your final answer.

(b) $f'(1) = \lim\limits_{h\to0} \dfrac{f(1+h)-f(1)}{h} = \lim\limits_{h\to0} \dfrac{\dfrac{-3(1+h)+1}{(1+h)+5} - \dfrac{-3(1)+1}{1+5}}{h} = \lim\limits_{h\to0}\left[\dfrac{-3-3h+1}{6+h} - \dfrac{-2}{6}\right]\cdot\dfrac{1}{h}$

$= \lim\limits_{h\to0}\left[\dfrac{-2-3h}{6+h}+\dfrac{1}{3}\right]\cdot\dfrac{1}{h} = \lim\limits_{h\to0}\left[\dfrac{-6-9h+6+h}{(6+h)3}\right]\dfrac{1}{h} = \lim\limits_{h\to0}\dfrac{-8h}{(6+h)3}\cdot\dfrac{1}{h} = \lim\limits_{h\to0}\dfrac{-8}{(6+h)3} = \dfrac{-8}{18} = -\dfrac{4}{9}$

(c)



Common Error: The asymptote for f' as $x\to\pm\infty$ is $y=0$, Not $y=-3$. The slope of the f graph is getting arbitrarily close to zero, Not -3

#2. (a) $L(t) = 3000 + 562.5t$

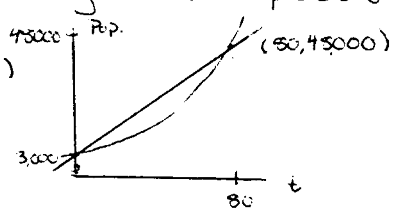
(b) $E(t) = 3000a^t$ we know $45000 = 3000a^{80}$ so $\left(\frac{45}{3}\right)^{1/80} = (a^{80})^{1/80}$

$(16)^{1/80} = a$

so $E(t) = 3000(16)^{t/80}$ OR $E(t) \approx 3000(1.035265)^t$

(c) Using the latter expression we see there's a $\approx 3.5265\%$ annual increase.

(d)



(e)(i) The slope of $E(t)$ at $t=0$ is much less than the slope of $L(t)$. (The latter has slope 526.5 and the slope of the former can be approximated numerically if you like - but the inequality is evident in the graph!)

(f)(ii) Both functions have the average rate of change $\frac{45{,}000-3000}{80}$. The average rate of change on the interval $[0,80]$ is the slope of the secant line, & $L(t)$ is that secant line.

(g) (i) For every t in $(0,80)$ the value of $L(t)$ > the value of $E(t)$ since the line lies above the exponential curve there.

#3. (a) $\lim\limits_{x\to\infty} g(x) = 2$

(b) $\lim\limits_{x\to-3} g(x) = -3$

(c) $\lim\limits_{x\to0} g(x) = 4$

(d) $\lim\limits_{x\to1^-} g(x) = -\infty$

(e) $\lim\limits_{x\to1} g(x)$ does not exist since $\lim\limits_{x\to1^-} g(x) = -\infty$ and $\lim\limits_{x\to1^+} g(x) = +\infty$

(f) $\lim\limits_{x\to\infty} g'(x) = 0$ (the curve approaches a horizontal line)

(g) $\lim\limits_{x\to-4^-} g'(x) = -5$ (the line connecting $(-5,0)$ and $(-4,$ has slope -5

(h) $\lim\limits_{x\to-4} g'(x)$ does not exist (since $\lim\limits_{x\to-4^-} g'(x) = -5$ and $\lim\limits_{x\to-4^+} g'(x)$ is some positive #.

#4.

a.) $A(48) = 320{,}000$ means that if the team wins 48 games in 1997 it can expect 320,000 in attendance the next year.

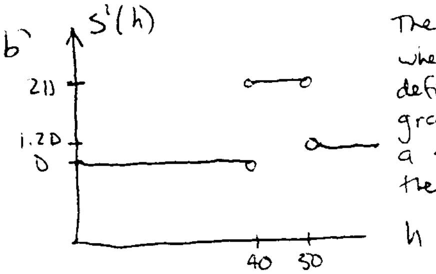
b.) $A'(90) = 24000$ means that when the team has won 90 games its expected 1998 attendance is increasing at 24000 fans per game won. In other words, winning a 91st game would bring in about 24000 more fans the next year.

#5.

a.)
$$S(h) = \begin{cases} Dh, & 0 \le h \le 40 \\ 40D + 2D(h-40), & 40 < h \le 50 \\ 60D + 1.2D(h-50), & 50 < h \end{cases}$$

Most mistakes dealt with $h > 40$. Let's look at $40 < h \le 50$. The trooper has already made $40D$ dollars for his/her first 40 hrs. The overtime pay is $2D$ dollars per hour above 40, i.e., $\underbrace{2D}_{\text{rate}}\underbrace{(h-40)}_{\text{time over 40}}$

b.)



The open dots are where S' is not defined because the graph of S has a sharp corner there.

$S(h)$ is piecewise linear, so $S'(h)$ is made up of horizontal lines.

#6.

a.) $v > 0$ when $t > -2 \Rightarrow s$ increasing there

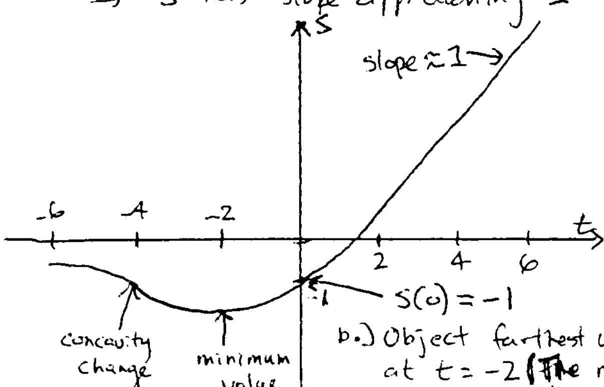
$v < 0$ when $t < -2 \Rightarrow s$ decreasing there

v incr when $t > -4 \Rightarrow s$ conc. up there

v decr when $t < -4 \Rightarrow s$ conc. down there

v approaches 1 as t increases on $[2, 6]$

$\Rightarrow s$ has slope approaching 1



b.) Object farthest west at $t = -2$ (The most negative s-value is there.)

#6.

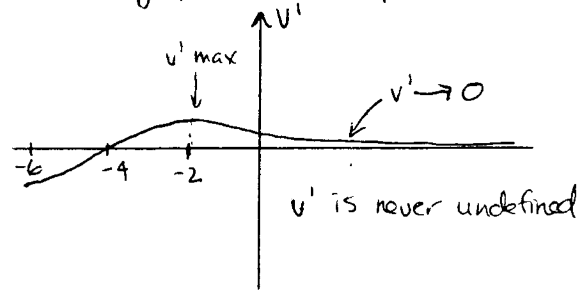
c.) v incr when $t > -4 \Rightarrow v'$ positive there

v decr when $t < -4 \Rightarrow v'$ negative there

v conc up when $t < -2 \Rightarrow v'$ incr there

v conc down when $t > -2 \Rightarrow v'$ decr there

v horiz asymptote $\Rightarrow v'$ approaches zero



#7.

a.) Panda's daily need: $\left(C \frac{\text{cal}}{\text{lb.}}\right)(P \text{ lbs.}) = CP \text{ cal}.$

There are N calories per S pounds sugarcane, so there are $\frac{N}{S} \frac{\text{cal}}{\text{lb sugarcane}}$.

Panda must eat $\dfrac{CP \text{ cal}}{\frac{N}{S}\frac{\text{cal}}{\text{lb. sugar}}} = \dfrac{CSP}{N}$ lbs. sugar

b.) Using the answer above, the pandas must eat $\frac{CSP}{N}$ and $\frac{CSQ}{N}$ pounds of sugar cane per day.

In W weeks, there are $7W$ days.

So, in W weeks, the two pandas need

$7W\left(\frac{CSP}{N} + \frac{CSQ}{N}\right)$ pounds sugarcane