MATHEMATICS Xa

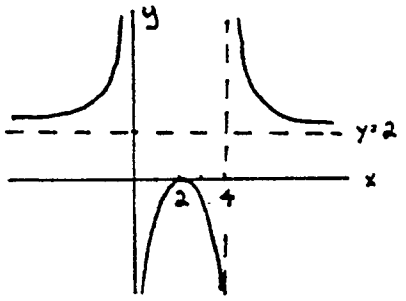
Second Examination

December 8, 1994

Part I. This part of the examination must be done without the aid of a calculator. when you have turned in Part I, you are free to use any sort of calculator you like.

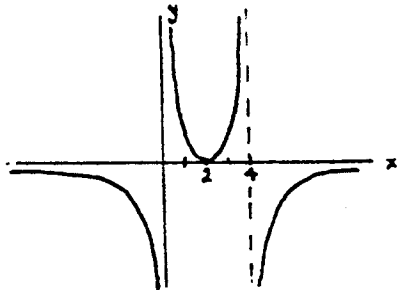
1. (12 points) Find possible equations for the following graphs. Please be sure your equations reflect the correct asymptotes (both horizontal and vertical), x - intercepts, and y - intercepts.

a)



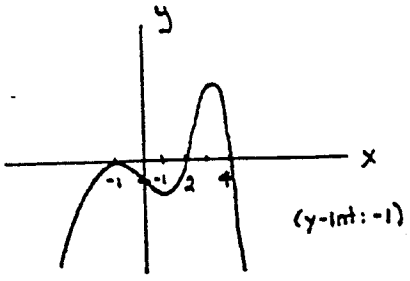
y = _____

b)



y = _____

c)



y = _____

2. (12 points) a) Graph $y = x^3 - 6x^2 + 9x$. Clearly label all x-intercepts, extrema, and points of inflection. Indicate your reasoning clearly and fill in the blanks below:

local maximum point(s): _____ local minimum point(s) : _____

point(s) of inflection: _____

b) Find the equation of the tangent line at the point of inflection. Draw this line on your graph.

3. (12 points) Graph $f(x) = 3 \ln x + 2 - 3x$ and answer the questions below:

a) What is the domain of f(x)?

b) Is there a local maximum point ? If so, what is it? _____

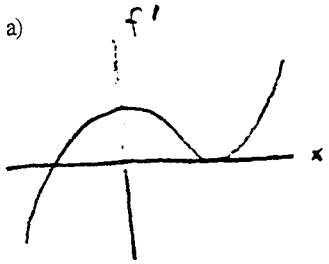
c) Is there a local minimum point? If so, what is it? _____

d) What is the global maximum **value**? How do you know this?

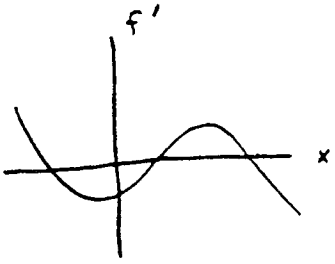
Part II:

1. (12 points) Given the graph of f ', draw a graph of f and f ". Please make your graph of f pass through the point (0,0).

a)



b)



2. (10 points) Differentiate

a) $y = 3 \ln(2x^5) + \ln (2/x)$

b) $y = 3^x - x^3 + 3^3$

3. (10 points) Solve for x:

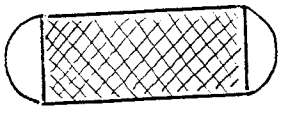
a) $3^{x^2+x} = 4^x$

b) $\ln(x^2) + 2 \ln (x^3) = 16$

4. (12 points) Hialeah Racetrack in Miami, Florida, is famous for the flamingos that live in the infield. It is reasonable, if you are concerned about the living quaters of these flamingos, to be interested in making the rectangular part of the infield as large as possible. It is this concern that motivates our problem. For the purpose of horseracing, the perimeter of the racetrack is fixed.

The racetrack consists of a rectangular region capped by two semicircles. The perimeter, P, is fixed. Find the dimensions of the track (find x and y) that will maximize the area of the rectangular part of the infield. The rectangular part of the infield is shaded in the picture below.

Please tell us how you know that the dimensions you found actually ***maximize*** the shaded infield area.



(For 1 point out of the 12): Suppose the perimeter is 7 furlongs, where a furlong is 1/8 of a mile. What is the area (in square miles) of the largest rectangular infield possible?

_____ square miles

5. (8 points) Suppose f is a continuous function whose domain is ($-\infty, \infty$) and suppose that $f'(-2) = 0$ and $f''(-2) < 0$. Circle **all** of the statements that are true. (If more than one statement is true then more than one statement should be circled.)

a) -2 is a critical point of f but it is neither a local maximum point nor a local minimum point.

b) f has a local maximum point at $x = -2$

c) f has a local minimum point at $x = -2$

d) f has a point of inflection at $x = -2$

e) f definitely has a global maximum at $x = -2$

f) f definitely has a global minimum at $x = -2$.

g) possibly f has a global maximum at $x = -2$, we don't have enough information to say for sure

h) possibly f has a global minimum at $x = -2$, we don't have enough information to say for sure

6. (12 points) Two populations of bugs - green bugs and yellow bugs - are growing exponentially.

a) (1 point)At time t=0 there are 800 green bugs, and the population increases by 6% every week. Write the function G(t), the number of green bugs after t weeks.

b) (5 points) The number of yellow bugs doubles every 5 weeks and is given by $Y(t) = 1000\ 2^{t/5}$, where t is measured in weeks. After how many weeks will the number of yellow bugs be twice as big as the number of green bugs. (Note: if your answer is, say, t = 10, then that would mean that Y(10) is twice as big as G(10).) Please give an exact answer and then give a decimal approximation.

c) (6 points) What is $Y'(5)$? Give a numerical answer and then interpret your answer in words (in terms of yellow bugs.)