Section 9.3 Applications of the Exponential Function

Problem 1.

(a) $C(t) = C_0\left(\frac{1}{2}\right)^{t/5730}$.

(b) $C(1993-137) = C(1856) = C_0\left(\frac{1}{2}\right)^{1856/5730} \approx (.7989)C_0$. In 1993, 79.98% of the original C_{14} remained.

(c) $C(t) = C_0\left(\frac{1}{2}\right)^{t/5730} = C_0\left(\frac{1}{2^{1/5730}}\right)^t \approx C_0(0.99988)^t$. Each year the amount of c_{14} in a deceased organism decreases by $100(1-0.99988)\% \approx 0.012\%$.

Problem 3.

(a) Let $G(t)$ be the mass (in mg) of the bacteria t hours after 7:00 AM. As $G(t)$ is growing exponentially and $G(0) = 12$, $G(t)$ is of the form $G(t) = 12a^t$. Now $G(5) = 14 \Rightarrow 12a^5 = 14 \Rightarrow a^5 = \frac{7}{6} \Rightarrow a = \left(\frac{7}{6}\right)^{1/5}$. Hence $G(t) = 12\left(\frac{7}{6}\right)^{t/5}$.

To find the mass after another five hours, substitute $t = 10$ into the formula for $G(t)$ to obtain

$$G(10) = 12\left(\frac{7}{6}\right)^{10/5} = 12\left(\frac{7}{6}\right)^2 = 12\left(\frac{49}{36}\right) = \frac{49}{3} \approx 16.3$$

Therefore, the mass of the population of the bacteria will be $\frac{49}{3}$ mg ≈ 16.3 mg after another five hours.

(b) As 7:00 PM corresponds to $t = 12$, the mass of the population at 7:00 PM is $G(12) = 12\left(\frac{7}{6}\right)^{12/5} \approx 17.4$.

(c) As 8:00 AM corresponds to $t = 1$, the mass of the population at 8:00 AM is $G(1) = 12\left(\frac{7}{6}\right)^{1/5} \approx 12.38$ mg.

Every hour, the mass is increasing by $100\left(1+\left(\frac{7}{6}\right)^{1/5}\right)\% \approx 3.13\%$. Each day, the mass is increasing by $100\left(1+\left(\frac{7}{6}\right)^{24/5}\right)\% \approx 109.58\%$.

Problem 10.

(a) After 10 years, Harvard's total return was $((1.111)^{10}-1)\cdot\% \approx 286.5\%$.

(b) Let $Y(t)$ be Yale's total return t years after 1985. Then $2.873 = Y(10) = x^{10} \Rightarrow x^{10} = 2.873 \Rightarrow x = 1.1113$. Hence, Yale's average annual return was 11.13%.

(c) Yale got the higher return on its investments. This can be seen by comparing average annual returns or the total returns over the ten-year period.

(d) For Harvard, $1.111^t = 2 \Rightarrow t = \frac{\ln 2}{\ln 1.111} \approx 6.585$. Thus the doubling time is 6.585 years.

For Yale, $1.1113^t = 2 \Rightarrow t = \frac{\ln 2}{\ln 1.1113} \approx 6.568$. Thus the doubling time is 6.568 years.

Problem 11.

(a) Given that $R(t) = 1010\cdot 2^{t/3}$, the initial population will double at the value for t for which $2020 = 1010\cdot 2^{t/3} \Leftrightarrow 2^{t/3} = 2 \Rightarrow \frac{t}{3} = 1 \Rightarrow t = 3$. The population will double after 3 years. The annual growth factor for $R(t)$ is $2^{1/3} \approx 1.2599$, which is an annual percent increase of 25.99%.

(b) Given that $S(t) = 3162(1.065)^t$, the initial population will double at the value for t for which $6324 = 3162(1.065)^t \Leftrightarrow (1.065)^t = 2 \Rightarrow t = \frac{\ln 2}{\ln 1.065} \approx 11$. (This value of t can be found using a graphing calculator.) The population will double after approximately 11 years. The annual growth factor for $s(t)$ is 1.065, which is an annual percent increase of 6.5%.

9.4

Problem 5.

$f'(x) = 3x^2 \cdot e^x + x^3 e^x$

Problem 6.

$f(x) = \dfrac{e^{2x}}{x} \Rightarrow f'(x) = \frac{2xe^{2x} - e^{2x}}{x^2} = \frac{e^{2x}}{x^2}(2x - 1)$

Problem 7.

$f'(x) = -3e^{-x} = -\frac{3}{e^x}$

Problem 8.

$f'(x) = \frac{(2x+1)(e^x+1) - (x^2+x)(e^x)}{(e^x+1)^2}$

Problem 9.

$f'(x) = 2e^{2x}(x^2 + 2x + 2) + e^{2x}(2x + 2) = 2e^{2x}(x^2 + 2x + 2 + x + 1) = 2e^{2x}(x^2 + 3x + 3)$