Section 10.2 Concavity and The Second Derivative

Problem 1.

(a) $f'(x) = 3x^2 - 6 = 0$, $3(x^2 - 2) = 0$, so $x^2 = 0 \Rightarrow x = \sqrt{2}$ and $x = -\sqrt{2}$ are critical points.

(b) Second derivative $f''(x) = 6x$. Since $f''(-\sqrt{2}) = -6\sqrt{2} < 0$, f has a local maximum at $x = -\sqrt{2}$. Since $f''(\sqrt{2}) = 6\sqrt{2} > 0$, f has a local minimum at $x = \sqrt{2}$

Problem 2.

(a) $f'(x) = -3x^2 + 3\pi^2 = -3(x^2 - \pi^2)$; $0 = f'(x) \Rightarrow x = -\pi$ and $x = \pi$ are the critical points.

(b) $f''(x) = -6x$. $f''(-\pi) > 0$, hence $x = -\pi$ is a local minimum point. $f''(\pi) < 0$, and hence $x = \pi$ is a local maximum point.

Problem 5.

(a) $f'(x) = 8x^3 + 64$; $0 = f'(x) \Rightarrow x = -2$ is the only critical point.

(b) $f''(x) = 24x^2$. As $f''(-2) > 0$, $x = -2$ is a local minimum point.

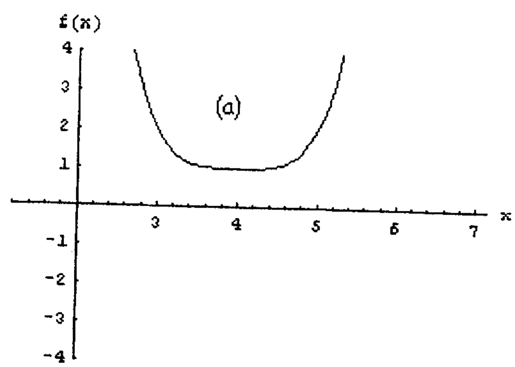
Problem 10.

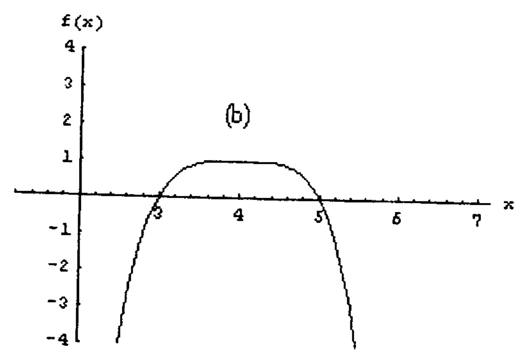
(a) $f'(x) = (1)e^x + xe^x - e^x = xe^x$; $0 = f'(x) \Rightarrow x = 0$ is the only critical point.

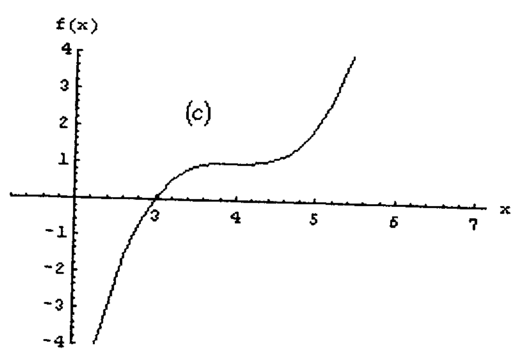
(b) $f''(x) = (1)e^x + xe^x = e^x + xe^x$. As $f''(0) > 0$, $x = 0$ is a local minimum point.

Problem 14.

(a) Local minimum: $f(x) = (x-4)^4 + 1$; (b) Local maximum: $f(x) = -(x-4)^4 + 1$; (c) Neither: $f(x) = (x-4)^3 + 1$.







Problem 16.

(a) (iii) and (vi)

(b) The critical points are $x = -8$, $x = -4$, and $x = -3$. The given information about the sign of h' implies that h has no local maxima and $x = -4$ is a local minimum point. The absolute maximum must occur at one (or both) endpoints, and the absolute minimum must occur at $x = -4$.