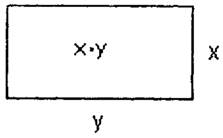
10.3

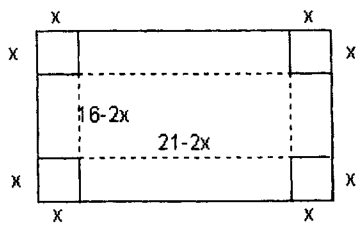
Problem 2.

Let L be the length of the fence, $L = 2x + 2y \quad \Rightarrow \quad \frac{L-2y}{2} = x$. The area of a garden as a function of



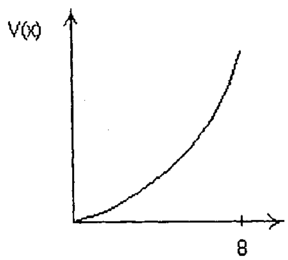
its length y is given by $A(y) = \frac{L-2y}{2} \cdot y = \frac{1}{2}(Ly - 2y^2) = -y^2 + \frac{L}{2}y$. To maximize, set $A'(y) = 0$ and solve for y: $-2y + \frac{L}{2} = 0 \;\Rightarrow\; y = \frac{L}{4}$. Now, $A''(y) = A''\left(\frac{L}{4}\right) = -2 < 0 \;\Rightarrow\; y = \frac{L}{4}$ is a maximum. If $y = \frac{L}{4} \;\Rightarrow\; x = \frac{L}{4} \;\Rightarrow\; x = y$, therefore we have a square.

Problem 4.



(a) $V(x) = (16-2x)(21-2x)x$. Domain $\begin{cases} 16 - 2x > 0 \\ 21 - 2x > 0 \\ x > 0 \end{cases} \Rightarrow \quad x \in (0, 8)$

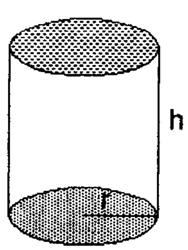
(b)



Problem 5.

From the Pythagorean Theorem, we have $h^2 + w^2 = 14^2$, and hence $h = \sqrt{196 - w^2}$. The strength of the beam is given by $S = kh^2w = k(196 - w^2)w = -kw^3 + 196kw$, where $k > 0$ is the proportionality constant. Now $S'(w) = -3kw^2 + 196k$. $S'(w) = 0 \Rightarrow w = \sqrt{\frac{196}{3}}$, and $S''(W) = -6kw < 0$ for all positive values of w. Hence the absolute maximum value of $S(w)$ is achieved when $w = \sqrt{\frac{196}{3}} \approx 8.08$ inches. At this width, the height $h = \sqrt{196 - \frac{196}{3}} = 14\sqrt{\frac{2}{3}} \approx 11.43$ inches.

Problem 13.



(a) $\pi r^2 \cdot h = 300 cm^3$

(b) $2 \cdot \pi r^2 + 2 \cdot \pi r^2 + 2\pi rh$

(c) $\pi r^2 \cdot h = 300 \quad \Rightarrow h = \frac{300}{\pi r^2}$, therefore if we plug in $\frac{300}{\pi r^2}$ instead of h into the equation from part (b) we get: $4\pi r^2 + 2\pi r \cdot \frac{300}{\pi r^2}$. So, cost of material is $C(r) = 4\pi r^2 + \frac{600}{r}$

(d) $r = \sqrt[3]{\frac{75}{\pi}}$

(e) $h = \frac{300}{\pi \cdot \frac{75^{2/3}}{\pi^{2/3}}} = \frac{300}{\sqrt[3]{\pi} \cdot \sqrt[3]{5625}} = \frac{300}{\sqrt[3]{5625\pi}}$