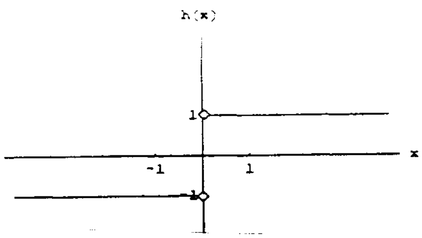
CHAPTER 3

Functions Working Together

Section 3.1 Combining Outputs

Problem 2.

The function $h(x) = f(x)g(x) = |x|\left(\frac{1}{x}\right) = \frac{|x|}{x}$ is undefined at $x = 0$ because $g(x)$ is undefined at $x = 0$. Note that $h(x) = 1$ for $x > 0$ and $h(x) = -1$ for $x < 0$.



Problem 6.

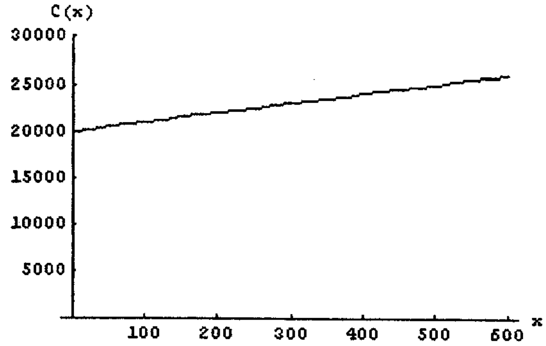
(a) At the rate of $1500\frac{gal}{h}$. The water flows in at the rate of $500\frac{gall}{h}$. Decreasing at the rate of $1000\frac{gall}{h}$.

(b) At 10:00 A.M. and 6:00 P.M.

(c) Between 10:00 A.M. and 4:00 P.M.

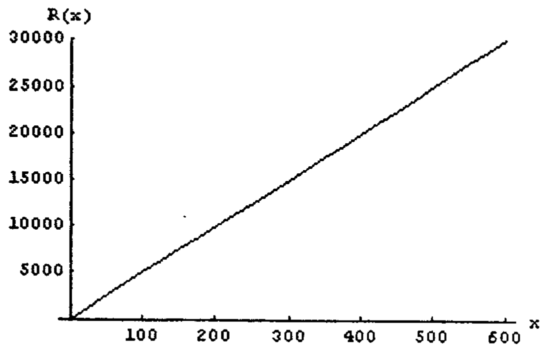
(d) Between 2:00 P.M and 4:00 P.M.

(a) $C(x) = 20,000 + 10x$.

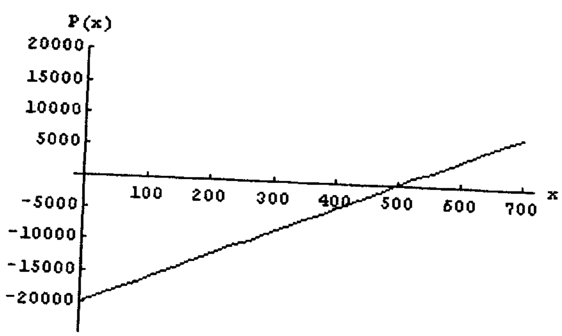


(b) The total cost is increasing by $\frac{\Delta C}{\Delta x} = \frac{C(x+\Delta x)-C(x)}{x+\Delta x - x} = \frac{(20,000+10(x+\Delta x)-(20,000+10x)}{\Delta x} = 10$ dollars per widget.

(c) $R(x) = 50x$.



(d) $P(x) = R(x) - C(x) = (50x) - (20,000 + 10x) = 40x - 20,000$.



(e) $P(400) = 40(400) - 20,000 = -4000$, which means that if 400 widgets are produced and sold then the company will lose \$4000. $P(700) = 40(700) - 20,000 = 8000$, which means that if 700 widgets are produced and sold then the company will have a profit of \$8000. $P(401) = 40(401) - 20,000 = -3960$ and $P(402) = 40(402) - 20,000 = -3920$. Profit increases by \$40 for each additional widget produced and sold. Thus, $\frac{\Delta P}{\Delta x} == \frac{P(x+\Delta x)-P(x)}{x+\Delta x - x} = \frac{((40(x+\Delta x)-20,000)-((40x-20,000)}{\Delta x} = 40$, which is constant for all values of x.

(f) $0 = P(x) = 40x - 20,000 \Leftrightarrow 40x = 20000 \Leftrightarrow x = 500$. The company must sell 500 widgets in order to break even.

(g) If the domain of $P(x)$ is the set of all integers where $0 \leq x \leq 1200$, the range of $P(x)$ is the set of all integers in the interval $= [40(0) - 20,000, 40(1200) - 20,000] = [-20000, 28000]$. Since the profit function is linear and has a positive slope, profit is maximized when production x is maximized; that is the company should sell all 1200 widgets to maximize profits.

3.2

Problem 3.

(a) $g(f(1)) = g(0) = -3$

(b) $f(g(0)) = f(-3) = 0$ (Note: $g(f(0)) = g(1) = -2$)

Problem 4.

(a) $f(f(2)) = f(-2) = 3$

(b) $f(f(1)) = f(0) = 1$

Problem 14.

(a) $f(x) + g(x) = (x-3) + (x^2 - 6x) = x^2 - 5x - 3$

(b) $f(x) - g(x) = (x-3) - (x^2 - 6x) = -x^2 + 7x - 3$

(c) $f(x)g(x) = (x-3)(x^2 - 6x) = x^3 - 6x^2 - 3x^2 + 18x = x^3 - 9x^2 + 18x$

(d) $f(g(x)) = f(x^2 - 6x) = x^2 - 6x - 3$

(e) $g(f(x)) = g(x-3) = (x-3)^2 - 6(x-3) = x^2 - 6x + 9 - 6x + 18 = x^2 - 12x + 27$

(f) $\frac{f(x)}{g(x)} = \frac{x-3}{x^2-6x}$

Problem 15.

(a) x-intercepts of $f(x)$ can be found by solving $f(x) = 0$. So, $x - 3 = 0 \Rightarrow x = 3$ is x-intercept.
For y-intercepts we set $x = 0$, so $y = 0 - 3 = -3$ is y-intercept.

(b) x-intercepts of $g(x)$: $x^2 - 6 = 0$ $x(x-6) = 0, \Rightarrow x = 0$ or $x = 6$.
For y-intercepts we set $x = 0$, so $g(0) = 0^2 - 0 \cdot 6 = 0$

(c) x-intercepts of $f(x) \cdot g(x)$: $x^3 - 9x^2 + 18x = 0$, $x(x^2 - 9x + 18) = 0$, $x(x-3)(x-6) = 0 \Rightarrow x = 0$ or $x = 3$ or $x = 6$.
For y-intercepts we set $x = 0$, so $f(0)g(0) = 0^3 - 9 \cdot 0^2 + 18 \cdot 0 = 0$ is y-intercept.

(d) $\frac{f(x)}{g(x)} = \frac{x-3}{x^2-6x}$, $D: x \neq 0$ and $x \neq 6$. x-intercepts: $\frac{x-3}{x^2-6x} = 0 \Leftrightarrow x - 3 = 0$, so $x = 3$ is x-intercept.
For y-intercepts we set $x = 0$, but 0 is excluded from the Domain, so there are no y-intercepts.

Problem 47.

(a) $f(f(2)) = f\left(\frac{1}{2} + 2\right) = f(\frac{5}{2}) = \frac{2}{5} + \frac{5}{2} = \frac{4}{10} + \frac{25}{10} = \frac{29}{10}$

(b) $g(g(-1)) = g\left(\frac{2(-1)}{(-1)^2+1}\right) = g(-1) = -1$

Problem 48.

(a) $f(g(x)) = f(\frac{2x}{x^2+1}) = \frac{x^2+1}{2x} + \frac{2x}{x^2+1}$

(b) $g(f(x)) = g(\frac{1}{x} + x) = g(\frac{x+1}{x}) = \frac{2 \cdot \frac{x+1}{x}}{(\frac{x+1}{x})^2+1}$