Math Xa
Worksheet—Finding Extrema
Solutions

Fall 2003

For each of the following functions,

- Find all critical points on the specified interval.
- Classify each critical point as a local maximum, a local minimum, an absolute maximum, an absolute minimum, or none of the preceding.
- If the function attains an absolute maximum or minimum, what is its maximum/minimum value?

1. $f(x) = x^3 - 3x + 2$ on $[0, 3]$.

   Solution. The derivative of f is

   $$f'(x) = 3x^2 - 3.$$

   If we set f' equal to zero, we see that the critical points are $x = -1$ and $x = 1$. We do not need to check $x = -1$, since it is not in the interval $[0, 3]$. Since

   $$\begin{aligned} f(0) &= 2 \\ f(1) &= 0 \\ f(3) &= 20, \end{aligned}$$

   we know that f attains its absolute minimum of 0 at $x = 1$ and its absolute maximum of 20 at $x = 3$.

2. $f(x) = 3x^4 - 8x^3 + 3$ on $(-\infty, \infty)$.

   Solution. The derivative of f is

   $$f'(x) = 12x^3 - 24x^2 = 12x^2(x - 2).$$

   If we set f' equal to zero, we see that the critical points are $x = 0$ and $x = 2$. By the First Derivative Test, a relative minimum occurs at $x = 2$ and $x = 0$ is neither a maximum or a minimum. Since $f(2) = -13$, and f is a fourth degree polynomial, f attains it absolute minimum at $x = 2$. That is,

   $$\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = \infty.$$

   There is no absolute maximum for f.

3. $f(x) = 3x^4 - 8x^3 + 3$ on $[-1, 1]$.

   Solution. Since

   $$\begin{aligned} f(-1) &= 14 \\ f(0) &= 3 \\ f(1) &= -2, \end{aligned}$$

   we have an absolute maximum of 14 at $x = -1$ and an absolute minimum of -2 at $x = 1$.

4. $f(x) = 3x^4 - 8x^3 + 3$ on $(0, 3)$.

   Solution. Since

   $$\begin{aligned} f(0) &= 3 \\ f(1) &= -2 \\ f(2) &= -13 \\ f(3) &= 30, \end{aligned}$$

   we have an absolute minimum of -13 at $x = 2$. Since we can never reach $x = 0$ or $x = 3$ on the interval $(0, 3)$, there is no absolute maximum.