Math Xa Lab 1
Algebra Skills
September 25, 2003

Name SOLUTIONS
Section Instructor
Collaborators

1.

(a) Simplify the expression $2^{\frac{3}{2}}2^{\frac{7}{2}}$. $2^{\frac{3}{2}} \cdot 2^{\frac{7}{2}} = 2^{\frac{3}{2}+\frac{7}{2}} = 2^{\frac{10}{2}} = 2^5 = 32$

(b) In general, if a, m, and n are numbers, what does $a^n a^m$ equal? $a^n a^m = a^{n+m}$

2. Each of the following equations is, in general, false. For each equation, give a numerical example that disproves the equation. Then simplify the left hand side of each equation correctly.

(a) $(a+b)^2 = a^2 + b^2$

Ex: $a = 2$, $b = 3$

$(a+b)^2 = (2+3)^2 = 5^2 = 25$
$a^2 + b^2 = 2^2 + 3^2 = 4+9 = 16$ ↙ not equal

$(a+b)^2 = (a+b)(a+b)$
$= a^2 + ab + ba + b^2$
$= a^2 + 2ab + b^2$

(b) $a^2 + a^2 = a^4$

Ex: $a = 3$

$a^2 + a^2 = 3^2 + 3^2 = 9 + 9 = 18$
$a^4 = 3^4 = 81$ ← not equal

$a^2 + a^2 = 2a^2$

(c) $\frac{1}{a} + \frac{1}{b} = \frac{1}{a+b}$

Ex: $a = 2$, $b = 3$

$1/a + 1/b = 1/2 + 1/3 = 3/6 + 2/6 = 5/6$
$\frac{1}{a+b} = \frac{1}{2+3} = \frac{1}{5}$ ← not equal

$\frac{1}{a} + \frac{1}{b} = \frac{1}{a} \cdot \frac{b}{b} + \frac{1}{b} \cdot \frac{a}{a}$
$= \frac{b}{ab} + \frac{a}{ab} = \frac{a+b}{ab}$

(d) $1 + \frac{a}{b} = \frac{a}{a+b}$

Ex: $a = 2$, $b = 3$

$1 + \frac{a}{b} = 1 + \frac{2}{3} = \frac{3}{3} + \frac{2}{3} = \frac{5}{3}$
$\frac{a}{a+b} = \frac{2}{2+3} = \frac{2}{5}$ ← not equal

$1 + \frac{a}{b} = \frac{b}{b} + \frac{a}{b} = \frac{a+b}{b}$

3. Factor the following expressions as much as possible.

(a) $x^2 - 16x + 64$

$= (x-8)(x-8) = (x-8)^2$

(b) $a^2 - 2ab + b^2$

$= (a-b)(a-b) = (a-b)^2$

(c) $4x^2 - 25$

$= (2x)^2 - 5^2 = (2x-5)(2x+5)$

(d) $a^2 - b^2 = (a-b)(a+b)$

(c) $2x^3 + 8x^2 - 42x = 2x(x^2 + 4x - 21)$

$= 2x(x+7)(x-3)$

4. Simplify the following expression.

$$\frac{1-\frac{1}{x}}{1-\frac{1}{x^2}}$$

$$\frac{1-\frac{1}{x}}{1-\frac{1}{x^2}} \cdot \frac{x^2}{x^2} = \frac{x^2(1-\frac{1}{x})}{x^2(1-\frac{1}{x^2})} = \frac{x^2 - \frac{x^2}{x}}{x^2 - \frac{x^2}{x^2}} = \frac{x^2 - x}{x^2 - 1}$$

$$= \frac{x(x-1)}{(x-1)(x+1)} = \boxed{\frac{x}{x+1}}$$

5. Solve the following equation for all values of x that satisfy it.

$$\frac{4}{4-x} = x$$

$$0 = \frac{4}{4-x} - x = \frac{4}{4-x} - x \cdot \frac{4-x}{4-x} = \frac{4}{4-x} - \frac{x(4-x)}{4-x}$$

$$= \frac{4 - x(4-x)}{4-x} = \frac{4 - 4x + x^2}{4-x} = \frac{x^2 - 4x + 4}{4-x}$$

$$\iff 0 = x^2 - 4x + 4 = (x-2)^2 \implies \boxed{x = 2}$$

6. Solve the following equation for all values of x that satisfy it.

$$5x^2 + x = 2$$

$5x^2 + x - 2 = 0$

Quadratic Formula:

$a = 5$

$b = 1$

$c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4(5)(-2)}}{2(5)}$$

$$= \boxed{\frac{-1 \pm \sqrt{41}}{10}}$$

7. Solve the following system of equations for all values of A and B that satisfy both equations.

$$\begin{aligned} A - 2B &= 7 \\ 4A + 3B &= 6 \end{aligned}$$

$A-2B=7$

$A=2B+7$ ↗

$4A+3B=6$

$4(2B+7)+3B=6$

$8B+28+3B=6$

$11B=-22$

$B=-2$

$A=2B+7$

$=2(-2)+7$

$=3$

$A=3, B=-2$

8. Express in words the values of x which satisfy the following inequality.

$$\frac{1}{x+4} \leq 3$$

$\frac{1}{x+4} \leq 3 \iff \frac{x+4}{1} \geq \frac{1}{3}$

$x+4 \geq 1/3$

$x \geq 1/3 - 4 = 1/3 - 12/3 = -11/3$

$x \geq -11/3$

all numbers greater than or equal to $-11/3$