Math Xa Fall 2003
Lab 2: Functions
September 30, 2003

Name SOLUTIONS
Section Instructor _____
Collaborators _____

You will need access to a graphing calculator to complete this lab. Make sure someone in your lab group has a graphing calculator. Also, all intervals you give as answers should be expressed in interval notation.

1. Let $f(x) = \sqrt{x^2 - 1}$.

(a) Evaluate $f(2)$ by hand. Then use your calculator to evaluate $f(2)$. Do your answers agree?

$f(2) = \sqrt{2^2-1} = \sqrt{4-1} = \sqrt{3}$

(b) Evaluate $f(\frac{5}{4})$ by hand. Then use your calculator to evaluate $f(\frac{5}{4})$. Do your answers agree?

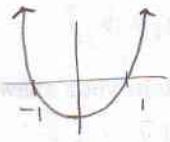
$f(\frac{5}{4}) = \sqrt{(\frac{5}{4})^2-1} = \sqrt{\frac{25}{16} - \frac{16}{16}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$

(c) Use your calculator to evaluate $f(\frac{1}{2})$. What does your calculator say when you do so? What does this mean?

"NONREAL ANS" — $f(1/2)$ is undefined

(d) What is the domain of f?

want $x^2 - 1 \geq 0$



$x^2 - 1 \geq 0$
when $x \leq -1$ or $x \geq 1$
$(-\infty, -1] \cup [1, \infty)$

(e) Assuming $t \geq -2$, find and simplify $f(\sqrt{t+2})$.

$f(\sqrt{t+2}) = \sqrt{(\sqrt{t+2})^2 - 1} = \sqrt{t+2-1} = \sqrt{t+1}$

2. Let $f(x) = x^3 - 400x$.

(a) Find the x-intercepts of f, if it has any, by hand.

$0 = x^3 - 400x = x(x^2 - 400) = x(x-20)(x+20)$

$x = -20, 0, 20$

(b) Use your answer to part (a) to graph f on your calculator in a window that shows its important features. Write down the minimum and maximum x and y values of the window you choose here.

$x: [-30, 30]$ $y: [-15000, 15000]$ (for example)

(c) On what exact interval(s) is f positive? Use your graph to assist you in answering this question.

$(-20, 0) \cup (20, \infty)$

(d) Use your calculator's trace feature to estimate the interval(s) on which f is increasing.

$(-\infty, -11.48) \cup (11.48, \infty)$

(e) Use your calculator's trace feature to estimate the interval(s) on which f is concave up.

$(0, \infty)$

3. Let $f(x) = \frac{1}{x-2}$.

(a) Find the x-intercepts of f, if it has any, by hand.

$0 = \frac{1}{x-2}$ ← can't happen! NONE

(b) Find the domain of f.

$(-\infty, 2) \cup (2, \infty)$

(c) Use your calculator to find $f(1.9)$, $f(1.99)$, and $f(1.999)$. What appears to be happening to the values of $f(x)$ as x approaches 2 from the left?

$f(1.9) = -10$
$f(1.99) = -100$
$f(1.999) = -1000$

As x approaches 2 from the left, $f(x)$ appears to decrease without bound.

(d) Graph f on your calculator in a window that shows its important features. Write down the minimum and maximum x and y values of the window you choose here.

$x: [-2, 6]$ $y: [-10, 10]$ (for example)

(e) Is f continuous or discontinuous at $x = 2$? Explain your answer.

Discontinuous because of the vertical asymptote at $x = 2$

4. Let $f(x) = |2x - 1|$.

(a) Find the x-intercepts of f, if it has any, by hand.

$|2x-1| = 0 \iff 2x - 1 = 0 \iff x = 1/2$

(b) Graph f on your calculator in a window that shows its important features. Write down the minimum and maximum x and y values of the window you choose here.

$x: [-10, 10]$ $y: [-10, 10]$ (for example)

(c) Find by hand the interval(s) on which $f(x) > 2$. Check your answer using your calculator's trace feature.

$|2x-1| > 2$

$2x - 1 < -2$ or $2x - 1 > 2$

$2x < -1$ $2x > 3$

$x < -1/2$ $x > 3/2$

$(-\infty, -1/2) \cup (3/2, \infty)$

5. Let $f(x) = x + 3$ and $g(x) = \dfrac{x^2 + 7x + 12}{x + 4}$.

(a) Find the domains of f and g.

domain of $f = (-\infty, \infty)$ domain of $g = (-\infty, -4) \cup (-4, \infty)$

(b) Are f and g the same function? Why or why not?

NO - f is defined at $x = -4$ and g is not

(c) Is g continuous or discontinuous at $x = -4$? Explain your answer.

Discontinuous because $g(-4)$ is undefined (a hole)

(d) Graph g on your calculator in a window that includes the point $(-4, -1)$. Does your calculator's graph convey your answer to part (c)? If so, describe how it does so.

Answers may vary depending on calculator settings.

6. Determine *algebraically* whether each of the following functions is even, odd, or neither.

(a) $f(x) = x + x^3 + x^5$

$f(-x) = -x + (-x)^3 + (-x)^5 = -x - x^3 - x^5$

$= -(x + x^3 + x^5) = -f(x)$ ODD

(b) $f(x) = x^2 + x^4$

$f(-x) = (-x)^2 + (-x)^4 = x^2 + x^4$ EVEN

$= f(x)$

(c) $f(x) = x + x^2$

$f(-x) = -x + (-x)^2 = -x + x^2$ NEITHER

(d) $f(x) = |x| + 4$

$f(-x) = |-x| + 4 = |x| + 4 = f(x)$ EVEN