

Math Xa—Lab 3
Composition of Functions
October 9, 2003

Name: _____
Section Instructor: _____
Collaborators: SOLUTIONS

1. If $f(x) = \sqrt{x}$, $g(x) = 1/x$, and $k(x) = 3x - 5$, find each of the following.

(a) $f(g(x))$

$$f(g(x)) = f\left(\frac{1}{x}\right) = \sqrt{\frac{1}{x}} = \frac{1}{\sqrt{x}}$$

(b) $g(f(x))$

$$g(f(x)) = g(\sqrt{x}) = \frac{1}{\sqrt{x}}$$

(c) $f(g(k(x)))$

$$f(g(k(x))) = f(g(3x-5)) = \frac{1}{\sqrt{3x-5}}$$

(d) $k(k(x))$

$$k(k(x)) = k(3x-5) = 3(3x-5) - 5$$

(e) $\frac{f(x+h) - f(x)}{h}$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \\ &= \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+h} + \sqrt{x}} \end{aligned}$$

2. How many ways can you find to decompose the function

$$h(x) = \left(\frac{\pi}{x^2-1}\right)^2 + 4\left(\frac{\pi}{x^2-1}\right) + 7$$

into functions f and g such that $f(g(x)) = h(x)$? For each possibility, list f and g .

$$\bullet \quad f(x) = x^2 + 4x + 7, \quad g(x) = \frac{\pi}{x^2-1}$$

$$\bullet \quad f(x) = \left(\frac{\pi}{x}\right)^2 + 4\left(\frac{\pi}{x}\right) + 7$$
$$g(x) = x^2 - 1$$

$$\bullet \quad f(x) = x + 7$$
$$g(x) = \left(\frac{\pi}{x^2-1}\right)^2 + 4\left(\frac{\pi}{x^2-1}\right)$$

There are many other possibilities.

3. Currency traders often move investments from one country to another in order to make a profit. The following table gives exchange rates for US dollars, Japanese yen, and the European Union's euro on April 23, 2001.

Amount invested	Dollars purchased	Yen purchased	European euro purchased
1 dollar	1.0000	121.17	1.1157
1 yen	0.0083	1.0000	0.0092
1 euro	0.8963	108.61	1.0000

For example, one US dollar purchases 121.17 Japanese yen or 1.1157 European euros. Similarly, one European euro purchases 108.61 Japanese yen or 0.8963 US dollars. Suppose

$$\begin{aligned} f(x) &= \text{Number of yen one can buy with } x \text{ dollars} \\ g(x) &= \text{Number of euros one can buy with } x \text{ dollars} \\ h(x) &= \text{Number of euros one can buy with } x \text{ yen} \end{aligned}$$

- (a) Find formulas for f , g , and h .

$$f(x) = 121.17x$$

$$g(x) = 0.8963x$$

$$h(x) = 0.0092x$$

- (b) Evaluate $h(f(1000))$ and interpret this in terms of currency.

$$\begin{aligned} h(f(1000)) &= h(121170) \\ &= 1114.764 \end{aligned}$$

If you trade \$1000 for yen and then trade the yen for euros, you can purchase 1114.76 euros. This is a rounded figure.

4. A function f has an inverse if there exists a function g such that

$$f(g(x)) = g(f(x)) = x.$$

We usually write f^{-1} instead of g .

Important! f^{-1} does not mean $1/f(x)$.

Let f be given by the following table.

x	0	1	2	3	4	5
$f(x)$	-1	0	7	26	63	124

Complete the following table.

x	-1	0	7	26	63	124	1330
$f^{-1}(x)$	0	1	2	3	4	5	11

Can you find algebraic formulas related to f and f^{-1} ?

$$f(x) = x^3 - 1$$

and

$$f^{-1}(x) = \sqrt[3]{x + 1}$$