Math Xa Fall 2003

Lab 8: Analysis of Extrema

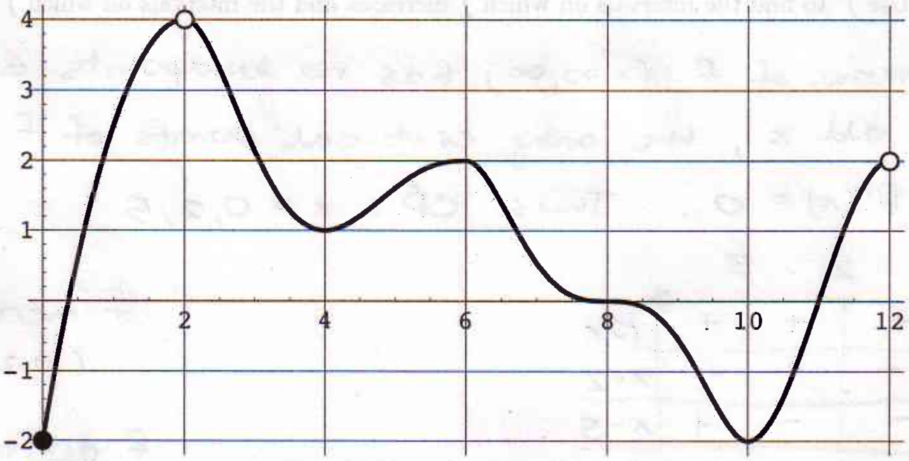
November 13, 2003

Name SOLUTIONS

Section Instructor

Collaborators

1. The graph of a function f is given below.



At what x-value(s) does f have a...

(a) local maximum? $x = 6$

(b) local minimum? $x = 4, 10$

(c) absolute maximum? none

(d) absolute minimum? $x = 0, 10$

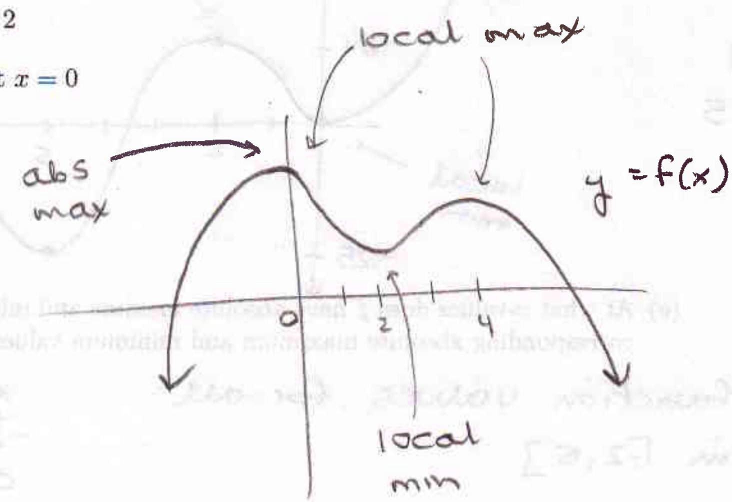
2. Sketch the graph of a single function f with domain $(-\infty, \infty)$ that satisfies *all* of the following conditions.

(a) f has local maxima at $x = 0$ and $x = 4$

(b) f has a local minimum at $x = 2$

(c) f has an absolute maximum at $x = 0$

(d) f has no absolute minimum



3. Let $f(x) = 60x^2 - 28x^3 + 3x^4$.

(a) Find $f'(x)$.

$$f'(x) = \boxed{120x - 84x^2 + 12x^3}$$
$$= 12x(10 - 7x + x^2)$$
$$= 12x(x^2 - 7x + 10) = 12x(x-5)(x-2)$$

(b) Use f' to find the intervals on which f increases and the intervals on which f decreases.

Since the domain of f $(-\infty, \infty)$ has no endpoints and $f'(x)$ is defined for all x, the only critical points of f are those for which $f'(x) = 0$. Thus CP: $x = 0, 2, 5$

	0	2	5	
−	+	+	+	$12x$
−	−	+	+	$x-2$
−	−	−	+	$x-5$
−	+	−	+	$f'(x) = 12x(x-2)(x-5)$
↘	↗	↘	↗	$f(x)$

f increases on $(0,2)$ and $(5,\infty)$

f decreases on $(-\infty, 0)$ and $(2,5)$

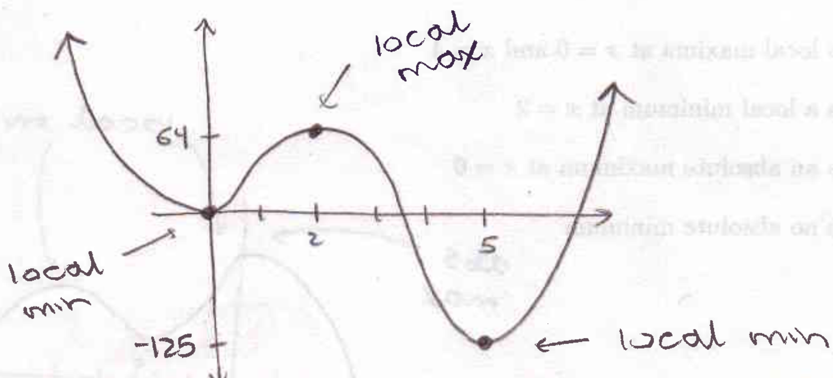
(c) Use the First Derivative Test to determine the x-values at which f has local minima and maxima.

local minimum : $x = 0, 5$

local maximum : $x = 2$

(d) Use your answers to parts (b) and (c) to sketch the graph of f. Plot and label the points that correspond to the local extrema of f.

x	$f(x)$
0	0
2	64
5	-125



(e) At what x-values does f have absolute maxima and minima on the interval $[-2, 6]$? What are the corresponding absolute maximum and minimum values?

Find function values for all CP in $[2,6]$

x	$f(x)$
-2	512
0	0
2	64
5	-125
6	0

- abs max at $x = -2$ max value of 512
- abs min at $x = 5$ min value of -125