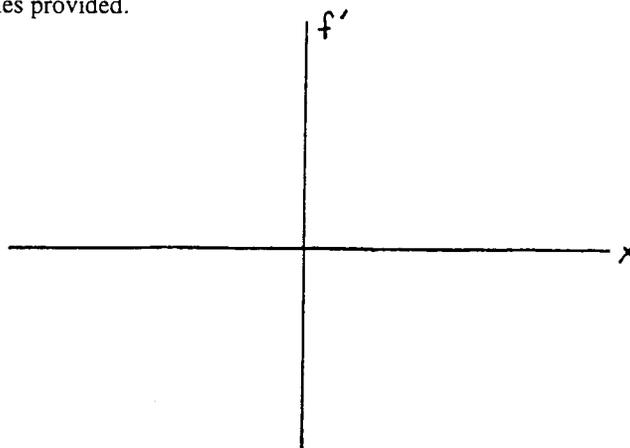
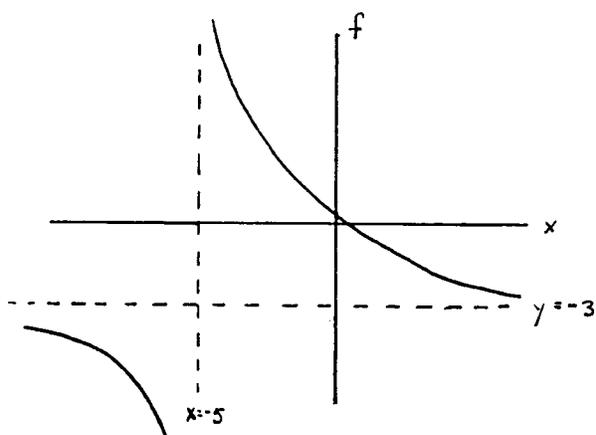


Mathematics Xa
 First Examination
 October 29, 1996

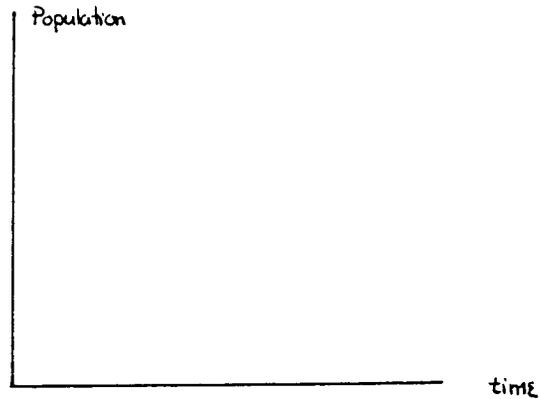
1. (17 points) Let f be the function $f(x) = \frac{-3x+1}{x+5}$.

- a) Use numerical methods to approximate $f'(1)$.
- b) Use the limit definition of derivative to compute $f'(1)$ exactly. In this part of the problem you are not being asked for a numerical approximation; you must produce an exact answer arrived at analytically. All your algebra must be explicitly demonstrated. (Hint: It is much simpler to calculate $f'(1)$ than $f'(x)$.)
 In part (a) we are interested in your answer. In part (b) of the problem we are interested in your write-up as well.
- c) The graph of f is given below. Sketch the graph of f' on the axes provided.



2. (23 points) The population of a certain town is growing over time. We will measure time t in years, with $t=0$ corresponding to 1800. The data we have indicates that in 1800 the population was 3000 and in 1880 the population was 48,000.
 In this problem we ask you to make two different population growth models, one linear and one exponential, and compare the two models.
- a) Assuming the population grows linearly, find an equation for the population as a function of time. Call this function $L(t)$.
- b) Assuming the population grows exponentially, find an equation for the population as a function of time. Call this function $E(t)$.
- c) Assuming the population grows exponentially (as in (b)), what is the annual percentage increase in population?

d) Sketch graphs of your answers to (a) and (b) labelling the coordinates of at least two points on the graph.



e) Which one of the following statements is true? Explain your reasoning.

- (i) $L'(0) > E'(0)$
- (ii) $L'(0) = E'(0)$
- (iii) $L'(0) < E'(0)$
- (iv) more information is needed to determine the relationship between $L'(0)$ and $E'(0)$.

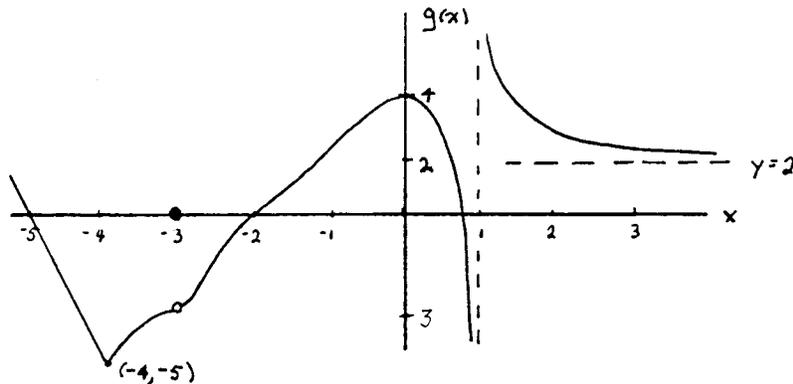
f) Select one of the following and explain your reasoning:

- (i) the average rate of change of $L(t)$ on $[0,80]$ $>$ the average rate of change of $E(t)$ on $[0,80]$.
- (ii) the average rate of change of $L(t)$ on $[0,80]$ $=$ the average rate of change of $E(t)$ on $[0,80]$.
- (iii) the average rate of change of $L(t)$ on $[0,80]$ $<$ the average rate of change of $E(t)$ on $[0,80]$.
- (iv) more information is needed to compare the average rates of change.

g) Suppose T is between 10 and 70 (corresponding to a year between 1810 and 1870). Select one of the following and explain your reasoning:

- (i) $L(T) > E(T)$
- (ii) $L(T) = E(T)$
- (iii) $L(T) < E(T)$
- (iv) more information is needed to determine the relationship between $L(T)$ and $E(T)$.

3. (16 points) The graph of $g(x)$ is given below. Evaluate the following:



a) $\lim_{x \rightarrow \infty} g(x)$

b) $\lim_{x \rightarrow -3} g(x)$

c) $\lim_{x \rightarrow 0} g(x)$

d) $\lim_{x \rightarrow 1^-} g(x)$

e) $\lim_{x \rightarrow 1} g(x)$

The next three questions ask about g' , NOT g .

f) $\lim_{x \rightarrow \infty} g'(x)$

g) $\lim_{x \rightarrow -4} g'(x)$

h) $\lim_{x \rightarrow -4} g'(x)$

4. (7 points) Let $A(w)$ be the expected 1998 attendance for a certain baseball team as a function of w , the number of games it wins in 1997. Interpret in plain English the meaning of each of the following.

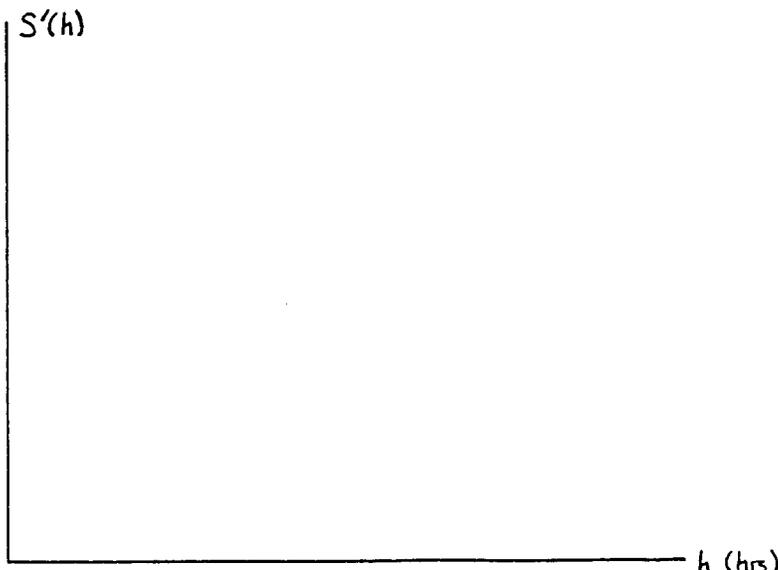
a) $A(48) = 320,000$

b) $A'(90) = 24,000$

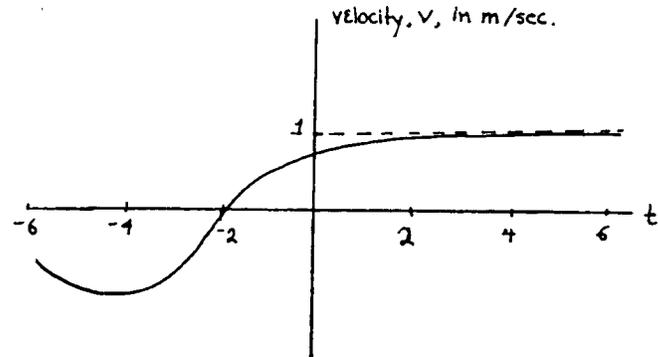
5. (12 points) Recent articles in the Boston Globe say that State troopers in Massachusetts are earning a lot of money by working overtime. Let's suppose the pay scale is modified so that it works in the following way. For the first 40 hours worked per week a trooper gets paid D dollars an hour. For the first 10 hours overtime (hours after the first 40) the trooper gets paid at an hourly rate that is double his or her ordinary hourly rate. Suppose that to discourage an excess of overtime, for overtime done in excess of 10 hours a week the hourly wage drops back to 120% of the officer's ordinary hourly rate.

- a) Write a function that gives S , the weekly salary of a state trooper, as a function of h , the number of hours worked per week. (Your answer should involve the constant D .)

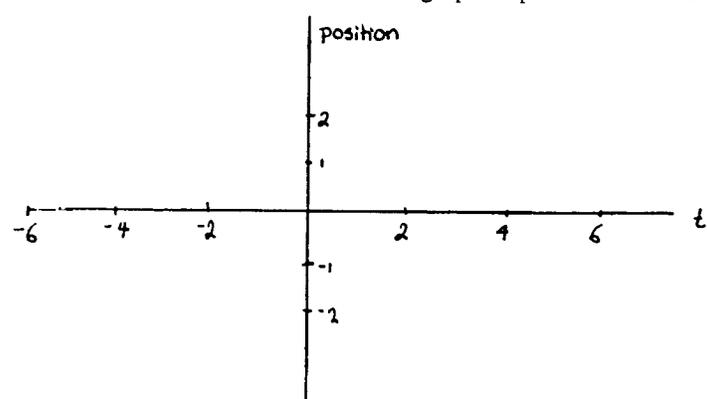
- b) Graph $S'(h)$.



6. (17 points) Below is a graph of the velocity of an object as a function of time t given in seconds. Assume that velocity is positive when the object is travelling east and negative when the object is travelling west.

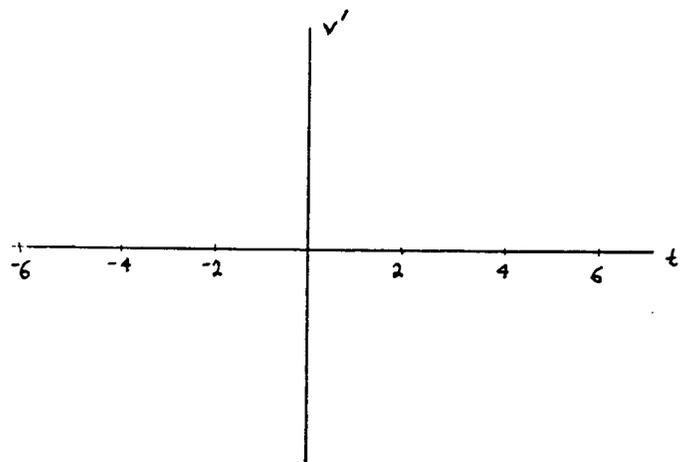


a) Suppose that at time $t=0$ the object is at position -1 . (Negative position values indicate that the object is west of position zero.) On the axes provided below, sketch the graph of position as a function of time.



b) When is the object farthest to the west? Explain your answer briefly.

c) On the set of axes provided below, graph acceleration as a function of time. (Recall that acceleration is v' .)



7. (8 points)

i) Giant Pandas eat sugar cane. On average, each day a Panda needs to take in C calories for every pound of the Panda's weight. If S pounds of sugar cane provide N calories, how many pounds of sugar cane does it take to support a Panda weighing P pounds for 1 day?

ii) How many pounds of sugar cane does it take to support a pair of Pandas, one weighing P pounds and the other weighing Q pounds, for a period of W weeks?