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MATHEMATICS Xa

Second Examination

December 11, 1995

1. (10 points) Solve for x: Please give exact answers.

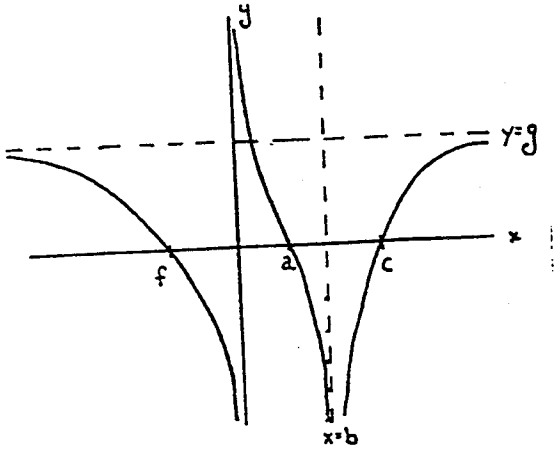
a) $(\ln x)^2 = 1$ b) $\log x^4 + \log\sqrt{x} = 3$ c) $(e^x)^3 = e^{x^3}$ d) $5(3^{2x+1}) = 2(4^x)$

2. (10 points) a) Find y ' if $y = 5\ln(7x^2) + 5e^{-3x} + \ln\left(\frac{5}{x^2}\right)$.

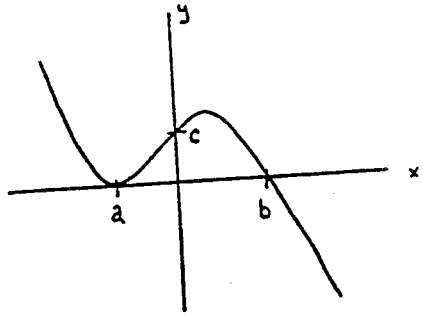
b) $f(x) = 5(3^{t/2})$ i) Find f '(x) ii) Find f "(1).

3. (10 points) Find possible equations to fit the following graphs: Be sure that your equation agrees with the x and y-intercepts and the vertical and horizontal asymptotes of the picture.

a)

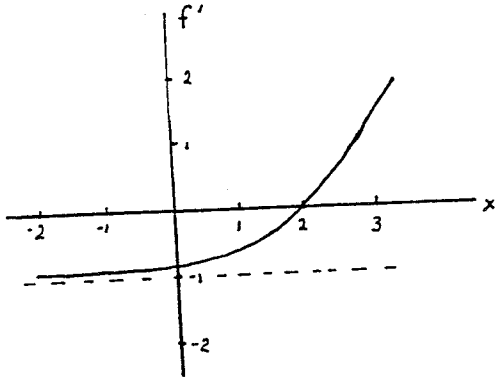


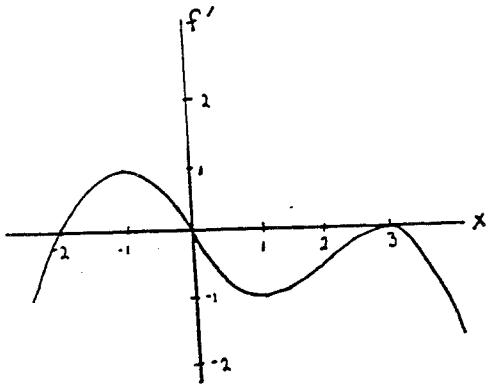
b)



4. (10 points) Given the graph of f ', draw a graph of f and of f ". Please have your graph of f pass through the point (0,0).

a)





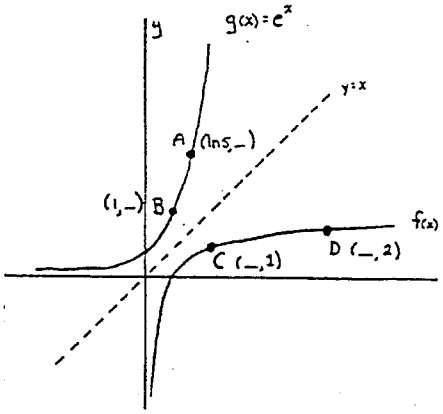
5. (10 points) For each part of this question, assume that f(x) is a function that has a stationary point at x = 5, i.e. f '(5) = 0. In all but one of the five cases given below, the piece of information proved is enough to classify this stationary point as either a local maximum, a local minimum, or a point of inflection. In one case there is insufficient data to classify the stationary point. If you can, classify the critical point; if there is not enough

information, write "insufficient data." Please provide a brief explanation for each of your answers.

a) $f''(x) = e^x$ b) $f(x) = (x-5)^2$ c) $f''(x) = (x-5)^3$

d) $f'(x) > 0$ for all $x \neq 5$ e) $f''(5) = 0$

6. (10 points) The following questions refer to the picture drawn. The picture is only a sketch. Your answers should be governed by your knowledge of the functions.



i) Fill in the blanks by supplying the indicated coordinates:

A: (ln5, ____)

B: (1, ____)

C: (____ , 1)

D: (____ , 2)

ii) Find the following:

a) the slope of the graph of g at point B, i.e. $g'(1)$

b) the second derivative of g at point A

c) the slope of the graph of f at point C

7. (10 points) A bear community living in the vicinity of a honey farm grows according to $P = P_0 e^{kt}$ where t is given in years. At time $t = 0$ there are 20 bears. Three years later there are 35 bears.

a) Find the growth equation.

b) When are there 50 bears?

c) At what rate is the population growing at $t = 0$? At $t = 3$?

8. (10 points) In this problem we will investigate the graph of $f(x) = 4 \ln x - (x-1)^2$. While you may use your calculator to check your work, you must explain your reasoning. "Because my calculator tells me so" is not an adequate explanation.

a) What is the domain of f?

b) Find and classify all critical points of f(x).

c) For what x is f increasing? Decreasing?

d) For what x is f ' increasing? decreasing?

e)What is the absolute minimum *value* taken on by the function.

f) Graph the function. What is the range of the function?

9. (5 points) Suppose $g(x) = 3\left(2^{-x+1}\right)$.

a)Find $g^{-1}(x)$. b) What is the domain of $g^{-1}(x)$? The range?

10. (15 points) We want to construct an open-top box with a square base. The volume must be 10 cubic feet. If the material for the bottom costs 15¢ per square foot and the material for the sides costs 6¢ per square foot, find the dimensions of the most economical box. Be sure to explain how you know that your dimensions actually minimize the cost. What *is* the cost of the most economical box?