Second Examination
Math Xa
December 9, 1996

1. (9 points) Find the derivative of each of the following functions.

a) $f(x) = \frac{-x^5}{\sqrt{5}} + \pi(e^x) + e^{\pi} + 3(5^x)$

b) $g(x) = \ln (4\sqrt{\pi}\ x^3\ e^{x/3})$

c) $h(x) = 3x\ e^{-4x}$

2. (12 points) Consider a function f whose domain is all positive numbers and whose derivative, f '(x), is given by $f'(x) = (x - 3)(\ln x - 2)$. The following questions refer to the function f itself.

a) What are the critical points of f?

b) Identify all local extrema of f. Which are local maxima and which are local minima? There are many ways to approach this problem. Please explain your reasoning clearly and completely.

c) If (1, 4) is a point on the graph of f, what is the equation of the tangent line to f at the point (1, 4)?

3. (12 points) Let $f(x) = \frac{x^3}{3} + x + 1$.

a) Where is f(x) increasing? decreasing?

b) Does f(x) have any points of inflection? If so, where/ Explain your reasoning briefly.

c) Is f(x) invertible without restricting the domain? Explain.

d) Graph f and f^{-1} on the same set of axes. (Feel free to use your calculator.) If necessary, restrict the domain. Label 3 points on the graph of f^{-1}. The coordinates of the points should be exact.

e) Evaluate $f^{-1}(f(0))$.

f) How many solutions are there to the equation $f(x) = k$? Does your answer depend on k?

g) Find the absolute maximum and minimum values of f on the interval [-2, 1].

4. (12 points) Postal regulations require that the sum of the girth and the length of a parcel sent by parcel post may be no more than 108 inches. Find the dimensions of the cylindrical package of greatest volume that can be mailed by parcel post.

   To earn full credit, you must give exact answers. Please show all your reasoning and present your work on the page in an organized way. Explain how you can be sure that the dimensions you found actually maximize the volume.

5. (9 points) Let P(D) be the number of passengers who will buy tickets on a certain airline flight as a function of D, the price (in dollars) of a seat. Suppose that the current price is A dollars per set and that this price consistently attracts an average of B passengers.

   Carefully interpret each of the following in words.

   a) $P(250) = 300$ b) $P'(275) = -1$ c) $P^{-1}(275) = 220$

   d) $P(A + 50) = 190$ e) $P^{-1}(B/2) = 3A$

6. (8 points) Solve for x in each of the following.

   a) $1 - 3\ln(2x + e) = 0$

   b) $\log x^2 = (\log x)^2$ Check your answer(s).

7. (12 points) A population of fish is currently 240 and is expected to double every 10 years. Let P(t) be the population t years from now.

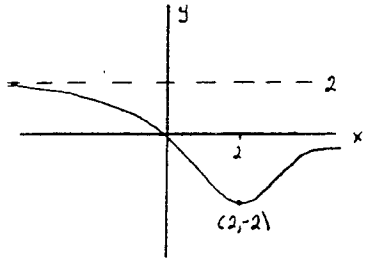
   a) Find an equation for P(t).

   b) When is the fish population 400? Give an exact answer and then give a decimal approximation.

   c) Write an equation whose solution tells us when the fish population is growing at a rate of 12 fish per year. (In other words, the solution should tell us when the instantaneous rate of change of the population is 12 fish per year.) Solve the equation exactly; then give a decimal approximation.

   d) Convert your equation from part (a) into an equation of the form $P(t) = C e^{kt}$, where C and k are the appropriate constants. Give a decimal approximation for k, using 3 decimals.

8. (6 points) Below is the graph of f(x).



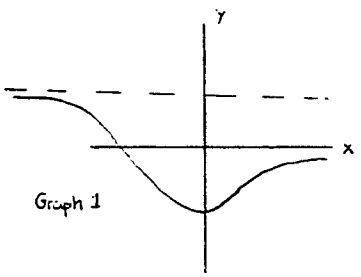
Fill in the blanks. Choose from the graphs at the bottom of the page.

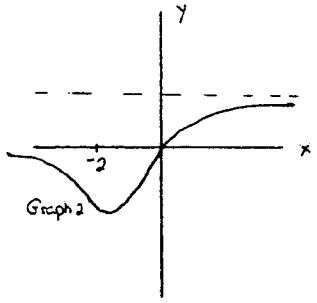
a) Graph # ____ is the graph of y = f(x-2).

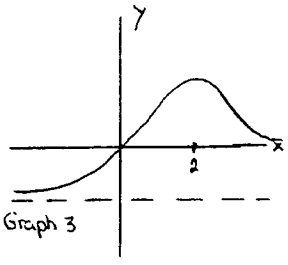
b) Graph #_____ is the graph of y = - f(x) + 2.

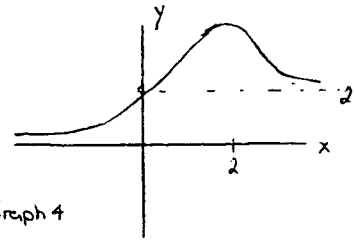
c) Graph #_____ is the graph of y = f(2x).

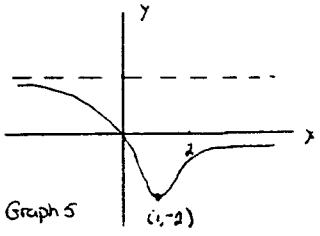
d) Graph #_____ is the graph of $y = f\left(\frac{x}{2}\right)$.

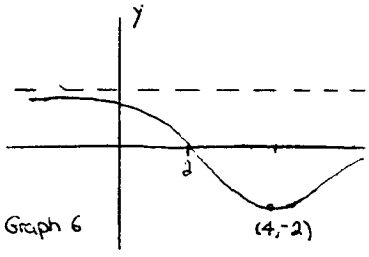


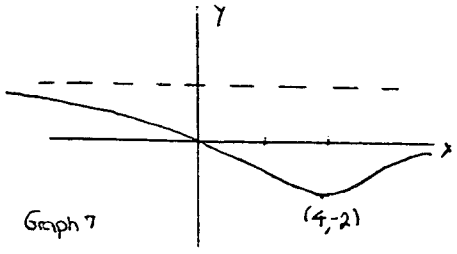


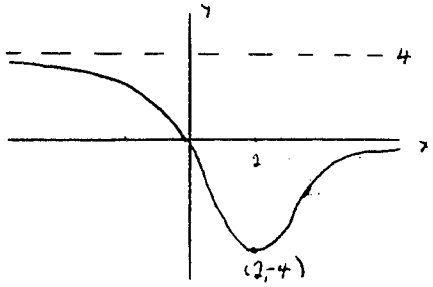






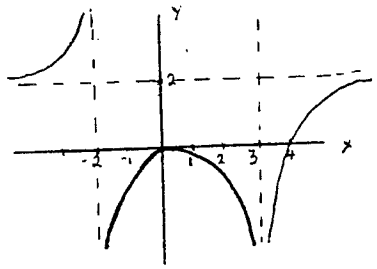






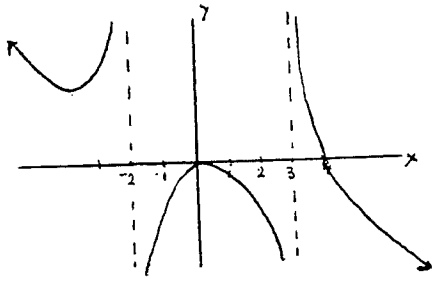
9. (10 points) Find an equation for each of the following graphs:

a)



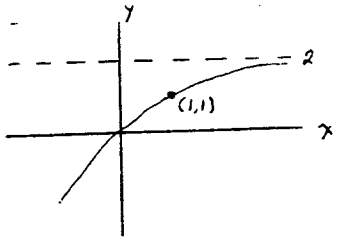
y = _____

b)



y = _____

c)



y = _____

10. (10 points)

i) Suppose P(x) is a polynomial of degree 5. Circle <u>all</u> of the following that are necessarily true.

a) P(x) has at least 1 root.

b) P(x) has no more than 4 roots.

c) The graph of P(x) has at least 1 turning point.

d) The graph of P(x) has no more than 4 turning points.

ii) Suppose P(x) is a polynomial of degree 5. <u>If</u> $P'(\pi) = 0$ and $P''(\pi) = 3$, then which <u>one</u> of the following statements is true?

a) P has a local minimum at $x = \pi$ but this local minimum is not an absolute minimum.
b) P has a local minimum at $x = \pi$ and this local minimum *may* be an absolute minimum.
c) P has a local maximum at $x = \pi$ but this local maximum is not an absolute maximum.
d) P has a local maximum at $x = \pi$ and this local maximum *may* be an absolute maximum.