Math Xa Fall 2003
Worksheet: Cubic Functions
November 19, 2003

1. For each of the following functions, graph the function and its derivative on a graphing calculator. Then determine the number of critical points of the function.

   (a) $f(x) = \frac{1}{3}x^3 + x + 1$

   (b) $g(x) = \frac{1}{3}x^3 + 1$

   (c) $h(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 1$

2. How many critical points can a generic cubic function $f(x) = ax^3 + bx^2 + cx + d$ have? Justify your answer.

3. How many local extrema can a cubic function have? Justify your answer.

4. How many inflection points can a cubic function have? Justify your answer.

5. If $f(x) = ax^3 + bx^2 + cx + d$, what are $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$?

6. How many x-intercepts can a cubic function have? Justify your answer.

7. Sketch the graph of a cubic function f that ...

   (a) ... has roots at $x = -5$, $x = 2$, and $x = 6$.

   (b) ... has roots at $x = -1$ and $x = 1$ only and satisfies $f(0) = 3$.

   (c) ... has a roots at $x = 3$ only and satisfies $\lim_{x \to \infty} f(x) = -\infty$.

   (d) ... has a local minimum at $x = -1$ and a local maximum at $x = 3$.

   (e) ... has an inflection point at $x = 3$.

8. Find an algebraic formula for each of the cubic functions described in Question 7.